THE SURJECTIVITY OF THE REDUCTION MAP FOR ALEXEEV'S SPACES

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Abstract. A matroid is a combinatorial object that can be thought of as a generalization of a spanning set of a vector space. As many other mathematical objects, it turned out that hyperplane arrangements have the matroid structure.

One can define a matroid using several different axiom systems such as independent sets, bases, span operator, flats, rank function, etc. I will show that two more descriptions of a matroid can be added that are characterized by edges of the associated convex polytope and quotient matroids, respectively, each of which leads us to a new combinatorial object, a *puzzle-piece*.

We will see moreover that the associated convex polytopes which are called *base polytopes* have a special gluing property which is distinguished from that of the polytopes that are just convex.

The gluing property tells us something about the surjectivity of the natural morphism, say the *reduction morphism*, defined between two moduli spaces $\overline{M}_\beta(k,n) \to \overline{M}_{\beta'}(k,n)$ with two *weights* $\beta > \beta'$, where $\overline{M}_\beta(k,n)$ is the moduli space of weighted stable hyperplane arrangements with rank k, which was generalized by Valery Alexeev from the Hasset's moduli space of curves of genus 0 with weighted n points with rank 2.

For Hassett's space, the reduction map is surjective for any weights $\beta \geq \beta'$, but for $k \geq 3$, the answer was expected to be "no" by the Mnev's universality theorem. In this talk, I will show that for $k = 3$ case, there is a counterexample to the surjectivity when $n = 10$, and that the map is surjective when $n = 4, 5, 6$.