CGP DERIVED SEMINAR

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1. February 13: Taesu Kim, an example of an A_∞ category

I'll talk about the Fukaya category, given a symplectic manifold (M,ω) which is oriented, compact, and spin and its spin Lagrangian submanifolds

$F \to M$ is the frame bundle whose fiber is frames of the tangent space. A frame is, given a basis of T_pM, the fiber F_p is the set of ordered orthonormal bases of T_pM. So I have $\iota c_1(TM) = 0$ and $\mu_L = 0$ in $H^1(L,\mathbb{Z})$ (this is the Maslov class) which are to give us a $\mathbb{Z}$-grading. Then we want $[\omega]\pi_2(M,L) = 0$ in order to avoid disk bubbling.

With this data we can define $F(M,\omega)$, the Fukaya category of (M,ω).

Let's talk about the Maslov class. Suppose that $2c_1 = 0$. This means that $(\wedge^n_{\mathbb{C}} T^*M)^{\otimes 2} \to M$ has a nonvanishing section, Θ. In local coordinates this looks like $v_1,\ldots,v_n \mapsto \Theta_p(v_1\wedge\cdots\wedge v_n \otimes v_1\wedge\cdots\wedge v_n)$, and then you can write

$$\frac{\Theta_p(\vec{v})\Theta_p(\vec{v})}{\Theta_p(\vec{v})\bar{\Theta}_p(\vec{v})}$$

which should be in S^1. So then we can define $\varphi_\Theta(p) \coloneqq \frac{\Theta^2(v)}{|\Theta^2(v)|}$. So for ℓ in $Gr(n)$, the Lagrangian plane, so inside T_pM we can find T_pL and for ℓ we assign this fraction above for arbitrarily chosen v. Then choose a lift of this map.

[long discussion]

So for each Lagrangian we have an index, and if they vanish, we can get some sort of loop by doing something in the cover at the intersection points. Then we get the Maslov index of a loop, and that's how we get the Maslov index for each intersection point.

[Long discussion]