IBS Center for Geometry and Physics

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(우 37673) 경상북도 포항시 남구 지곡로 127번길 79 기초과학연구원 포스텍캠퍼스 기하학수리물리연구단

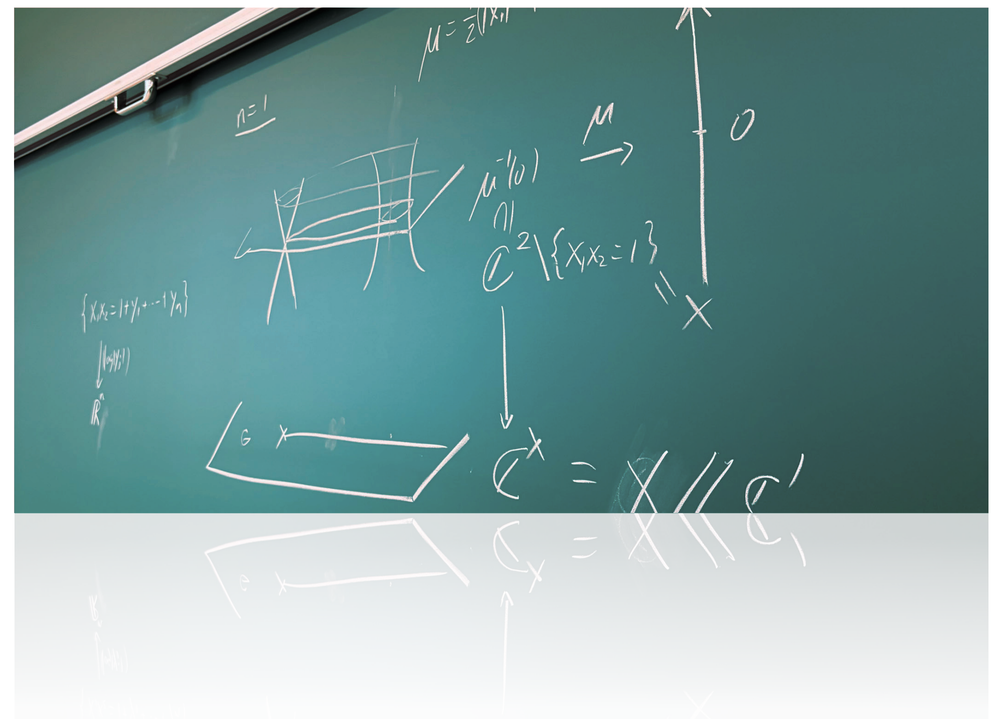
CGP Walk 2025

IBS Center for Geometry and Physics

IBS Center for Geometry and Physics

CGP Walk

Beyond the horizon



The Sixth Issue
2025

기초과학연구원
Institute for Basic Science

IBS Center for Geometry and Physics

CGP Walk

Beyond the horizon

The Sixth Issue
2025

기초과학연구원
Institute for Basic Science

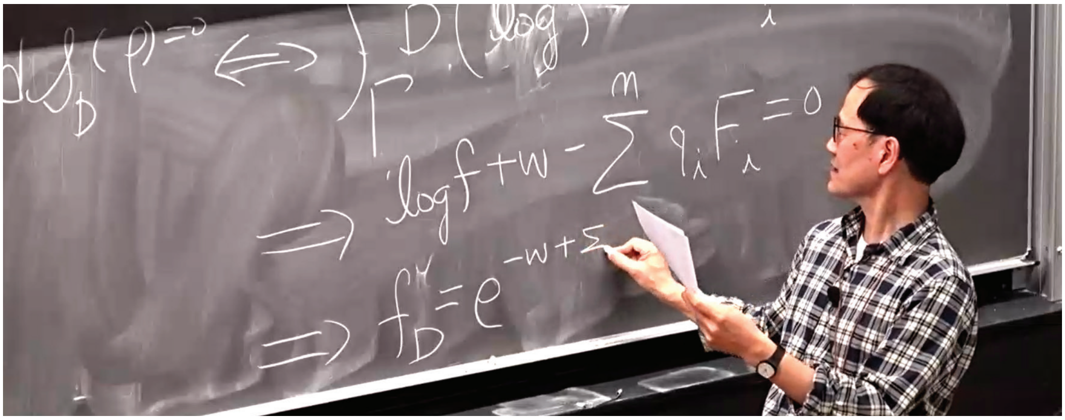
Contents

Director's Note

The year 2025 was the 100th year since Werner Heisenberg revealed his new ideas on physics in a letter to Wolfgang Pauli on July 9th 1925. Many anniversary events and workshops worldwide, such as the Helgoland 2025 workshop, celebrated 100 years of quantum mechanics and marked this major scientific milestone, in the very island of Helgoland where Heisenberg made the first formulation of Quantum Theory. The United Nations celebrated the centenary of quantum science. The Simons Center for Geometry and Physics at Stony Brook is also organizing a one-semester thematic program (October 2026 - January 2027) to celebrate this milestone.

Since its birth, quantum theory not only revolutionized the physics but also created many branches of new mathematics, operator algebras, spectral theory, probability, representation theory and so on. Although it is 100 years old, it is still very up-to-date. It keeps delivering new puzzles, experimental ideas and quantum technologies as well as new challenging mathematics of the world of quantum probability, quantum information theory, and gravity; welcome to *the world of quantum gravity!*

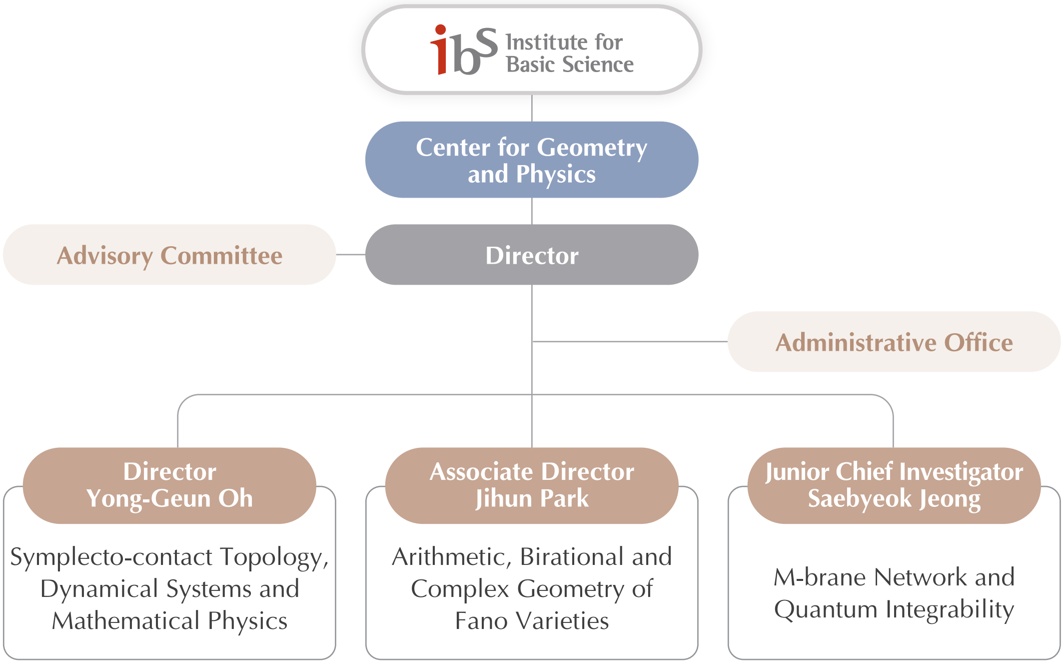
I always find remarkable to see how the laws of nature, the blueprint of God's creation, is crystalized into human's language through scientists' experiments and investigations followed by mathematician's abstraction of them into new mathematical worlds and then applied back to scientist's further deepening of human's understanding of the nature. On one hand, I am happy to participate in this men's highest activity of opening up the secrets of nature and go beyond the intellectual horizon of the humanity. On the other hand, in this fast-changing quantum uncertainty world, I feel both hope and anxiety on the future of the humanity.



Organization

One arching research theme of the CGP is to promote interaction between symplectic geometry, algebraic geometry and mathematical physics in the study of symplectic topology and homological mirror symmetry and their applications to theoretical physics.

The CGP is organized into multi research groups, each of which comprises a senior scholar and several researchers whose areas of expertise and interest overlap synergistically.



Research Groups

As of December 2025

Symplecto-contact Topology, Dynamical Systems and Mathematical Physics (PI: Yong-Geun Oh)

Symplectic topology and mirror symmetry

Yong-Geun Oh | Symplectic topology, Hamiltonian dynamics and mirror symmetry

Sam Bardwell-Evans | Symplectic geometry, Moduli spaces, Pseudoholomorphic curves and discs, Mirror symmetry

Yan-Lung Li | Symplectic geometry, Lagrangian Floer Theory, 2d and 3d mirror symmetry

Jihyeon Lee | Differential geometry, geometric analysis

Jaewon Chang | Symplectic Geometry, Lagrangian Floer Theory, Mirror symmetry

Jonghyeon Ahn | Symplectic geometry and Hamiltonian dynamics

Ju Tan | Symplectic Geometry, and Mirror Symmetry

Florian Michael Zeiser | Poisson geometry, Lie theory and foliations

Seong Jin Lee | Quantum topology, Knots and links, Machine learning applications to low dimensional topology

Mathematical physics

Alexander Alexandrov | Mathematical physics, random matrix models, integrable systems, enumerative geometry

Norton Lee | Supersymmetry, Integrable Systems, Quantum Field Theories, Mathematical Physics

Dmytro Voloshyn | Mathematical physics, cluster algebras, Poisson geometry, quantum groups, integrable systems

Jorge Gigante Valcarcel | Metric-affine geometry, gauge theories of gravity, cosmology and black hole physics

Arithmetic, Birational and Complex Geometry of Fano Varieties (PI: Jihun Park)

Jihun Park | Birational and complex geometry of Fano varieties

Igor Krylov | Birational Geometry

Jaekwan Jeon | Deformations of rational surface singularities and related topics in symplectic geometry

Luca Rizzi | Complex Algebraic Geometry, families of varieties, deformations and Torelli-type problems, Hermitian metrics

Sukjoo Lee | Mirror symmetry and Hodge theory

Mykola Pochekai | Mirror symmetry and Hodge theory

M-brane Network and Quantum Integrability (PI: Saebyeok Jeong)

Saebyeok Jeong | M-brane Network and Quantum Integrability

CGP Advisory Committee

The CGP Advisory Committee consists of eight distinguished scholars from Korea and abroad. The committee meets once a year and provides advice and input on the operations of the Center.

The current members of the Advisory Committee are (as of December 2025):

Alexander Givental
Professor at University of California, Berkeley

Sergei Gukov
Professor at California Institute of Technology

Bo-Hae Im
Professor at Korea Advanced Institute of Science and Technology (KAIST)

Jae-Hun Jung
Professor at Pohang University of Science and Technology (POSTECH)

Ludmil Katzarkov
Professor at University of Miami & Institute of Mathematics and Informatics (Bulgarian Academy of Sciences)

Young-Hoon Kiem
Professor at Korea Institute for Advanced Study (KIAS)
Director of June E Huh Center for Mathematical Challenges

Sijong Kwak
President at Korean Mathematical Society (KMS)
Professor at Korea Advanced Institute of Science and Technology (KAIST)

Herman Verlinde
Professor at Princeton University

Scientific Activities

Statistics

5 conferences
4 colloquium talks and 49 seminar talks
2 Intensive lecture series

CGP at a Glance

MAR	• New Research Member **Jihyun Lee** • Event **Pohang Workshop on Birational Geometry** (March 24-28)
MAY	• Event **BICMR-IBSCGP-NewUU Joint Conference on Geometry, Algebra and Mathematical Physics** (May 21-27)
JUN	• New Research Member **Seongjin Lee** • Event **Conference on Integrable Systems and Related Areas** (June 23-27)
AUG	• New Research Member **Jaewon Chang, Jonghyun Ahn**
SEP	• New Research Member **Saebyeok Jeong** (JCI), **Ju Tan**
OCT	• New Research Member **Florian Micheal Zeiser**
NOV	• Event **RIMS & IBS-CGP Joint Workshop** (November 19-21)
DEC	• Event **Conference on Representations of Quivers in Mathematics and String Theory** (December 8-12)

Conferences

Pohang Workshop on Birational Geometry;

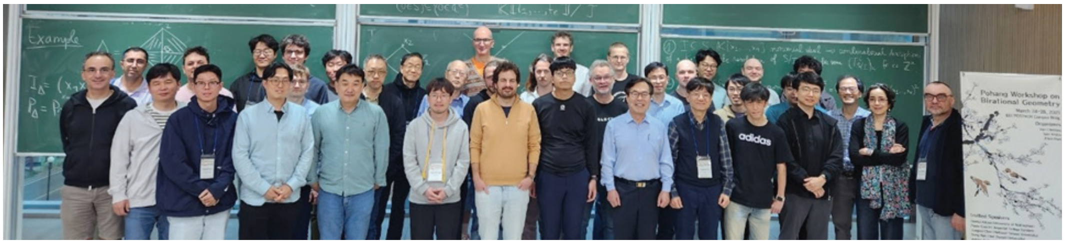
March 24-28, 2025

Organizers

- Ivan Cheltsov (The University of Edinburgh)
- Igor Krylov (IBS Center for Geometry and Physics)
- Jihun Park (IBS Center for Geometry and Physics & POSTECH)

Invited Speakers

- Hamid Abban (University of Nottingham)
- Paolo Cascini (Imperial College London)
- Jungkai Chen (National Taiwan University)
- Sung Rak Choi (Yonsei University)
- Adrien Dubouloz (University of Poitiers)
- Kento Fujita (Osaka University)
- Tiago Duarte Guerreiro (Paris-Saclay University)
- Jun-Muk Hwang (Institute for Basic Science)
- Anne-Sophie Kaloghiros (Brunel University London)
- Takashi Kishimoto (Saitama University)
- Yongnam Lee (Institute for Basic Science)
- Constantin Loginov (Steklov Mathematical Institute)
- Frederic Mangolte (Aix-Marseille University)
- Andrea Petracci (University of Bologna)
- Yuri Prokhorov (Steklov Mathematical Institute)
- Taro Sano (Kobe University)
- Constantin Shramov (Steklov Mathematical Institute)
- Robert Smiech (University of Edinburgh)
- Luca Tasin (University of Milan)



BICMR-IBSCGP-NewUU Joint Conference on Geometry, Algebra and Mathematical Physics;

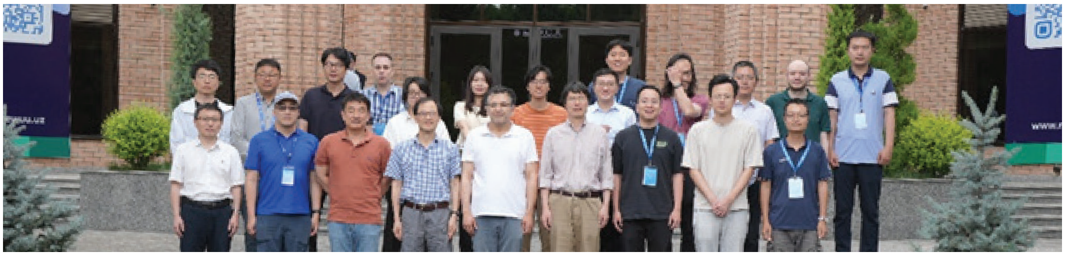
May 21-27, 2025

Organizing Committee

- Bahodir Ahmedov (New Uzbekistan Univerisity, Uzbekistan)
- Xiaojun Chen (New Uzbekistan Univerisity, Uzbekistan & Sichuan University, China)
- Yong-Geun Oh (IBS Center for Geometry and Physics & POSTECH, Korea)
- Rustam Turdibaev (New Uzbekistan University, Uzbekistan)
- Qizheng Yin (BICMR, Peking University, China)
- Bakhtiyor Yuldashev (New Uzbekistan Univerisity, Uzbekistan)

Invited Speakers

- Yalong Cao (Chinese Academy of Sciences)
- Guodu Chen (Shanghai Jiao Tong University)
- Xiaojun Chen (New Uzbekistan University/ Sichuan University)
- Farkhod Eshmatov (New Uzbekistan University)
- Huijun Fan (Wuhan University)
- Isroil A. Ikromov (Uzbekistan Academy of Sciences)
- Jingjun Han (Fudan University)
- Yoosik Kim (Pusan National University)
- Igor Krylov (IBS Center for Geometry and Physics)
- Sukjoo Lee (IBS Center for Geometry and Physics)
- Zhiyuan Li (Fudan University)
- Yongqi Liang (University of Science and Technology of China)
- Wenfei Liu (Xiamen University)
- Wenhao Ou (Chinese Academy of Sciences)
- Jinhyung Park (KAIST)
- Emanuel Scheidegger (Peking University)
- Guo Chuan Thiang (Peking University)
- Zhiyu Tian (Peking University)
- Jorge Gigante Valcarcel (IBS Center for Geometry and Physics)
- Joonyeong Won (Ewha Womans University)
- Xiaomeng Xu (Peking University)
- Di Yang (University of Science and Technology of China)
- Song Yang (Tianjin University)
- Bin Zhang (Sichuan University)



Conference on Integrable Systems and Related Areas;

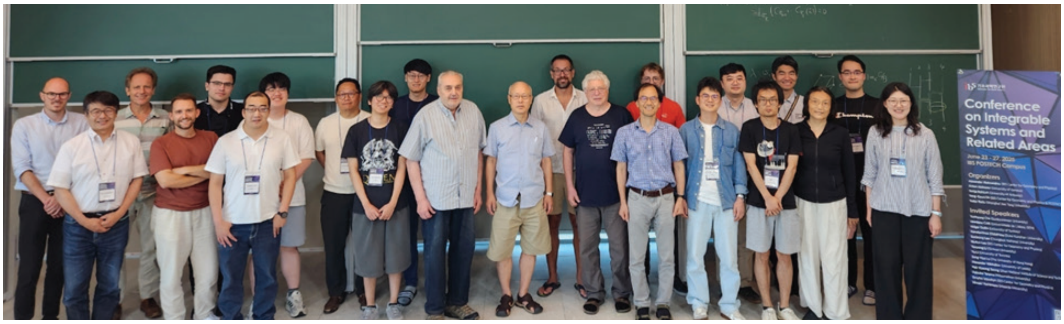
June 23-27, 2025

Organizers

- Alexander Aleksandrov (IBS Center for Geometry and Physics)
- Anton Alekseev (University of Geneva)
- Sonja Hohloch (University of Antwerp)
- Yong-Geun Oh (IBS Center for Geometry and Physics & POSTECH)
- Tudor Ratiu (Shanghai Jiao Tong University)

Invited Speakers

- Yunhyung Cho (Sungkyunkwan University)
- Giordano Cotti (Universidade de Lisboa, GEM)
- Holger Dullin (University of Sydney)
- Konstantinos Efstathiou (Duke Kunshan University)
- Eunjeong Lee (Chungbuk National University)
- Norton Lee (IBS Center for Geometry and Physics)
- Yanpeng Li (Sichuan University)
- Yu Li (University of Toronto)
- Jiang-Hua Lu (The University of Hong Kong)
- Alexander Mikhailov (University of Leeds)
- Rak-Kyeong Seong
  (Ulsan National Institute of Science and Technology)
- Daisuke Tarama (Ritsumeikan University)
- Dmytro Voloshyn (IBS Center for Geometry and Physics)
- Hiroaki Yoshimura (Waseda University)



RIMS & IBS-CGP Joint Workshop,

November 19-21, 2025

Organizers

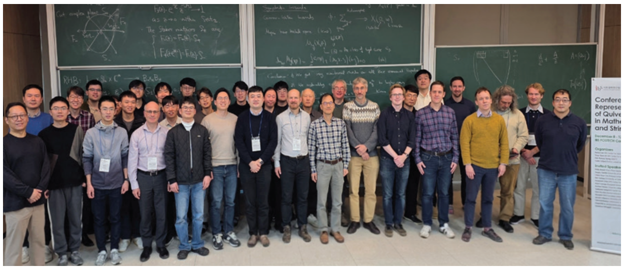
- Kaoru Ono (RIMS, Kyoto University)
- Yong-Geun Oh (IBS Center for Geometry and Physics & POSTECH)

Invited Speakers

- Wenmin Gong (Beijing Normal University)
- Axel Husin (RIMS, Kyoto University)
- Kei Irie (RIMS, Kyoto University)
- Jungsoo Kang (Seoul National University)
- Dogancan Karabas (Temple University)
- Jongmyeong Kim (QSMS, Seoul National University)
- Sukjoo Lee (IBS-CGP)
- Tian-Jun Li (University of Minnesota)
- Hiro Lee Tanaka (Texas State University)
- Weiwei Wu (Zhejiang University)

Conference on Representations of Quivers in Mathematics and String Theory;

December 8-12, 2025

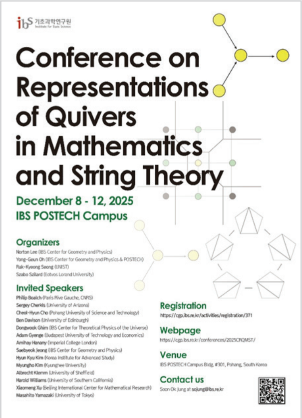


Organizers

- Norton Lee (IBS Center for Geometry and Physics)
- Yong-Geun Oh (IBS Center for Geometry and Physics & POSTECH)
- Rak-Kyeong Seong (UNIST)
- Szabo Szilard (Eotvos Lorand University)

Invited Speakers

- Philip Boalch (Paris Rive Gauche, CNRS)
- Sergey Cherkis (University of Arizona)
- Cheol-Hyun Cho (Pohang University of Science and Technology)
- Ben Davison (University of Edinburgh)
- Dongwook Ghim (IBS Center for Theoretical Physics of the Universe)
- Adam Gyenge (Budapest University of Technology and Economics)
- Amihay Hanany (Imperial College London)
- Saebyeok Jeong (IBS Center for Geometry and Physics)
- Hyun Kyu Kim (Korea Institute for Advanced Study)
- Myungho Kim (Kyunghee University)
- Albrecht Klemm (University of Sheffield)
- Harold Williams (University of Southern California)
- Xiaomeng Xu (Beijing International Center for Mathematical Research)
- Masahito Yamazaki (University of Tokyo)



Talks

Symplectic configurations: a homological and computer-aided approach
Weimin Chen (University of Massachusetts at Amherst)
December 31, 2025

Constructing small symplectic 4-manifolds via contact gluing and some applications
Weimin Chen (University of Massachusetts at Amherst)
December 30, 2025

Coherent Langrangian classes
Dongwook Choa (Chungbuk National University)December 29, 2025

Seminar Series in General Relativity
Jorge Gigante Valcarcel (IBS-CGP)
December 19, 2025

On Symplectic Fillings
Zhengyi Zhou (Academy of Mathematics and Systems Science)
December 15, 17, 18 2025

Extended Chern-Simons Theories and Free Differential Algebras
Sebastian Salgago (Bernardo O'Higgins University)
December 12, 2025

Path integral derivations of K-theoretic Donaldson's invariant
Hee Yeon Kim (KAIST)
December 5, 2025

Gamma conjecture II for two-step flag varities $F\ell_{1,n-1;n}$
Jiayu Song (Sun Yat-Sen University)
December 5, 2025

Introduction to Atoms IV
Kyoung-Seog Lee (POSTECH)
December 4, 2025

S^1-equivariant relative symplectic cohomology and relative symplectic capacities
Jong Hyeon Ahn (IBS-CGP)
December 1, 2025

[IBS-CGP&POSTECH-Math Colloquium] Lambda-invariant among KR-modules
Sejin Oh (Sungkyunkwan University)
November 28, 2025

Introduction to Atoms II
Jaekwan Jeon (IBS-CGP)
November 18, 27, 2025

Localized Mirror Construction and Hecke Correspondences II
Ju Tan (IBS-CGP)
November 17, 2025

Seminar Series in General Relativity
Jorge Gigante Valcarcel (IBS-CGP)
November 14, 2025

Elliptic stable envelopes from Gauge Theory
Nafiz Ishtiaque (Shanghai Institute for Mathematics and Interdisciplinary Sciences)
November 12, 2025

Introduction to Atoms I
Sukjoo Lee (IBS-CGP)
November 11, 2025

Normal forms and cohomolgy in Poisson Geometry
Florian Micheal Zeiser (IBS-CGP)
November 10, 2025

Generalized Chern-Simons Theory as a String Theory
Philsang Yoo (Seoul National University)
November 5, 2025

Stable envelope for critical Loci
Yehao Zhou (Shanghai Institute for Mathematics and Interdisciplinary Sciences)
November 5, 2025

Hecke Correspondences from Localized Mirror Construction
Ju Tan (IBS-CGP)
November 3, 2025

[IBS-CGP&POSTECH-Math Colloquium] Excellent Morse Functions (joint with Lisa Traynor)
Hiro Lee Tanaka (Texas State University)
October 31, 2025

Wrapped Floer Theory and Liouville Sectors
Jaewon Chang (IBS-CGP)
October 27, 2025

A Brief Introduction to Wall-Crossing in Floer theory
Sam Bardwell-Evans (IBS-CGP)
October 13, 20, 2025

Seminar Series in General Relativity
Jorge Gigante Valcarcel (IBS-CGP)
October 10, 17, 2025

Poisson-Lie groups and cluster algebras
Dmytro Voloshyn (IBS-CGP)
October 1, 2025

Cluster integrable system on a chessboard
Norton Lee (IBS-CGP)
September 24, 2025

Automorphisms of affine varieties: flexibility and unipotent group actions
Alexander Perepechko (HSE University)
September 23, 2025

Equivariant Lagrangian correspondence and its applications
Yan-Lung Li (IBS-CGP)
September 22, 2025

Seminar Series in General Relativity
Jorge Gigante Valcarcel (IBS-CGP)
September 19, 2025

Introduction to Cluster Algebras 2
Dmytro Voloshyn (IBS-CGP)
September 17, 2025

A Brief Introduction to Global Kuranishi Charts in Floer theory
Sam Bardwell-Evans (IBS-CGP)
September 8, 15, 2025

Introduction to Cluster Algebras
Dmytro Voloshyn (IBS-CGP)
September 9, 2025

Seminar Series in General Relativity
Jorge Gigante Valcarcel (IBS-CGP)
August 8, 29, 2025

XY Swap Duality in Topological Recursion
Maxim Kazarian (Higher School of Economics, Skoltech)
August 1, 2025

Fully simple maps and X-Y duality in topological recursion
Boris Bychkov (University of Haifa)
July 30, 2025

Seminar Series in General Relativity
Jorge Gigante Valcarcel (IBS-CGP)
July 4, 25, 2025

The Noether inequality for threefolds and three moduli spaces with minimal volumes
Yong Hu (Shanghai Jiao Tong University)
July 22, 2025

Shifted Contact Structures on Differentiable Stacks
Luca Vitagliano (University of Salerno)
July 17, 2025

Symplectic ellipsoid embeddings, singular plane curves, and scattering diagrams
Kyler Siegel (University of Southern California)
July 10, 2025

Mirror of Log Calabi-Yau Surfaces
Hyunbin Kim (Yonsei University)
July 7, 2025

Seminar Series in General Relativity
Jorge Gigante Valcarcel (IBS-CGP)
June 20, 2025

[IBS CGP-POSTECH Math Colloquium] Complex continued fractions and beyond
Seonhee Lim (Seoul National University)
May 16, 2025

[Intensive Lecture Series] Path to instanton partition function: ADHM construction on noncommutative geometry I -III
Norton Lee (IBS-CGP)
May 7-9, 2025

Effective gonality theorem on weightone syzygies of algebraic curves
Jinhyung Park (KAIST)
April 29, 2025

Hodge Mirror Symmetry for Fano Manifolds
Sukjoo Lee (IBS-CGP)
April 16, 2025

Weak del Pezzo surfaces with big tangent bundles
Jeong Seop Kim (KIAS)
April 15, 2025

Gushel Mukai fourfolds and flops
Marco Rampazzo (University of Antwerp)
April 8, 2025

Geometricaspects of the categorical resolution of nodal Gushel-Mukai fourfold
Kacper Grzelakowski (University of Lodz)
April 8, 2025

[IBS CGP-POSTECH Math Colloquium] Complex surfaces with minimal Betti numbers
Jonghae Keum (KIAS)
April 4, 2025

Equivariant Ulrich bundles on rational homogeneous varieties of Picard number one
Kyeong Dong Park (Gyeongsang National University)
February 25, 2025

Deformations of sandwiched surface singularities and the minimal model program
Dongsoo Shin (Chungnam Nataional University)
February 20, 2025

[Intensive Lecture Series] Iterates of Symplectomorphisms, Floer Homology, and p-adic Analysis I -II
Yusuf Baris Kartal (University of Edinburgh)
February 17-18, 2025

Colored Jones Polynomials on 4-Plat closures
Philip Choi (Seoul National University)
January 23, 2025

Bohr-Sommerfeld Surgeries on Lagrangian Submanifolds
Soham Chanda (University of Southern California)
January 9, 2025

Positive recurrence of Brownian motion on surface with pinched negative curvature
Jaelin Kim (Alfred Renyi Institute of Mathematics)
January 8, 2025

Visitors

[December]

Heeyeon Kim (KAIST)
Jiayu Song (Sun Yat-sen University)
Weimin Chen (University of Massachusetts-Amherst)
Dongwook Choa (Chungbuk University)
Zhengyi Zhou (Academy of Mathematics and Systems Science)
Sebastián Salgado (Universidad Bernardo O'higgins)
Hyun Kyu Kim (KIAS)
Myungho Kim (Kyunghee University)
Cheol-Hyun Cho (POSTECH)
Kyoung-Seog Lee (POSTECH)
Rak-Kyeong Seong (UNIST)
Szabo Szilard (Eotvos Lorand University)
Amihay Hanany (Imperial College London)
Masahito Yamazaki (University of Tokyo)
Harold Williams (University of Southern California)
Ben Davison (University of Edinburgh)
Adam Gyenge (Budapest University of Technology and Economics)
Philip Boalch (Paris Rive Gauche, CNRS)
Albrecht Klemm (University of Sheffield)
Xiaomeng Xu (Beijing International Center for Mathematical Research)
Sergey Cherkis (University of Arizona)
Eunjeong Lee (Chungbuk National University)
Donggun Lee (IBS Center for Complex Geometry)
Dongwook Ghim (IBS Center for Complex Geometry)

[November]

Sejin Oh (Sungkyunkwan University)
Nafiz Ishtiaque (Shanghai Institute for Mathematics and Interdsciplinary Sciences)
Philsang Yoo (Seoul National University)
Yehao Zhou (Shanghai Institute for Mathematics and Interdisciplinary Sciences)
Sijong Kwak (KAIST)
Bo-Hae Im (KAIST)
Jaehoon Jeong (POSTECH)
Sergei Gukov (California Institute of Technology)
Ludmil Katzarkov (University of Miami)

[October]

Hiro Lee Tanaka (Texas State University)

[September]

Alexander Perepechko (HSE University)
Dongseon Hwang (IBS Center for Complex Geometry)

[July]

Sergey Shadrin (University of Amsterdam)
Petr Dunin-Barkowski (National Research University Higher School of Economics, Moscow)
Maxim Kazarian (National Research University Higher School of Economics, Moscow)
Boris Bychkov (University of Haifa)
Yong Hu (Shanghai Jiao Tong University)
Luca Vitagliano (University of Salerno)
Kyler Siegel (University of Southern California)
Hyunbin Kim (Yonsei University))

[June]
Tudor Ratiu (Shanghai Jiao Tong University)
Daisuke Tarama (Ritsumeikan University)
Hiroaki Yoshimura (Waseda University)
Yu Li (University of Toronto)
Yanpeng Li (Sichuan University)
Jiang-Hua Lu (The University of Hong Kong)
Alexander Mikhailov (University of Leeds)
Holger Dullin (University of Sydney)
Giordano Cotti (Universidade de Lisboa, GEM)
Konstantinos Efstathiou (Duke Kunshan University)
Rak-Kyeong Seong (UNIST)
Yunhyung Cho (Sunkyunkwan University)
Eunjeong Lee (Chungbuk National University)
Xiaomeng Xu (Peking University)
Anton Izosimov (University of Arizona)
Anton Alekseev (University of Geneva)
Sonja Hohloch (University of Antwerp)
Peter Crooks (Utah State University)

[May]
Seonhee Lim (Seoul National University)

[April]
Marco Rampazzo (University of Antwerp)
Kacper Grzelakowski (University of Łódź)
Jeong-Seop Kim (KIAS)
Jinhyung Park (KAIST)
Jonghae Geum (KIAS)

[March]
Ivan Cheltsov (University of Edinburgh)
Hamid Abban (University of Nottingham)
Paolo Cascini (Imperial College London)
Jungkai Chen (National Taiwan University)
Sung Rak Choi (Yonsei University)
Adrien Dubouloz (University of Poitiers)
Kento Fujita (Osaka University)
Tiago Duarte Guerreiro (Paris-Saclay University)
Anne-Sophie Kaloghiros (Brunel University London)
Takashi Kishimoto (Saitama University)
Constantin Loginov (Steklov Mathematical Institute)
Frederic Mangolte (Aix-Marseille University)
Andrea Petracci (University of Bologna)
Yuri Prokhorov (Steklov Mathematical Institute)
Taro Sano (Kobe University)
Constantin Shramov (Steklov Mathematical Institute)
Luca Tasin (University of Milan)
Robert Smiech (University of Edinburgh)
Yat-Hin Suen (National Cheng Kung University)
Jun-Muk Hwang (IBS Center for Complex Geometry)
Yongnam Lee (IBS Center for Complex Geometry)

[February]
Sungmun Cho (POSTECH)
Yusuf Baris Kartal (University of Edinburgh)
Kyeong-Dong Park (Kyeongsang National University)
Dongsoo Shin (Chungnam National University)

[January]
Sungmun Cho (POSTECH)
Soham Chanda (University of Southern California)
Phillip Choi (Seoul National University)
Jaelin Kim (Alfrűd Rűnyi Institute of Mathematics)

MOUs

The CGP has signed several MOUs for active research collaborations and academic exchanges with the mathematics community.

Beijing International Center for Mathematical Research (BICMR), China

November 2015-November 2026 (renewed in 2021)

- BICMR & IBS-CGP & NewUU Joint Conference on Geometry, Algebra and Mathematical Physics (May 21-27, 2025 @ New Uzbekistan University)
- 4th IBS-CGP & BICMR Joint Conference on Gromov-Witten Theory and Related Topics (August 26-30, 2024 @ Jeju Uni Hotel)
- 3rd BICMR&IBS-CGP Joint Symplectic Geometry Workshop (September 24-26, 2019 @ POSTECH)
- Silk Road Geometry Conference 2018 (June 4–8, 2018 @ Gцkova Geometry/Topology Institute)
- 2nd BICMR&IBS-CGP Joint Symplectic Geometry Workshop (September 18-22, 2017 @ BICMR)
- 1st BICMR&IBS-CGP Joint Symplectic Geometry Workshop (October 31-November 4, 2016 @ Jeju KAL Hotel)

Research Institute for Mathematical Sciences (RIMS), Japan

August 2017-July 2028 (renewed in 2025)

- RIMS & IBS-CGP Joint Workshop (November 19-21, 2025 @ Kyoto University)
- RIMS-IBSCGP Conference on Recent Developments in Symplectic Topology (November 4-8, 2024 @ POSTECH)
- 2021 Pacific Rim Complex & Symplectic Geometry Conference (July 12-16, 2021 @ Online)
- RIMS & IBS-CGP Joint Symplectic Geometry Workshop (December 2-4, 2019 @ Kyoto University)
- Wall-crossing Formula, Open Gromov-Witten Invariants and Related Areas (October 29-31, 2018 @ POSTECH)
- Pacific Rim Complex-Symplectic Geometry Conference (July 31-August 4, 2017 @ POSTECH)

Mathematical Research Institute (MATRIX), Australia

December 2018-November 2026 (renewed in 2021)

- MATRIX-IBSCGP workshop on Symplectic and Low-dimensional Topology (June 3-14, 2024 @ POSTECH)
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The Centre de recherches mathйmatiques (CRM), Canada

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Research Highlight

Scattering diagrams from holomorphic discs in log Calabi-Yau surfaces

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From string theory to mathematics

To discuss the notion of scattering diagrams as they appear in the setting of mathematical mirror symmetry (and in particular in my recent paper with Cheung, Hong, and Lin [1]), it is helpful to begin with some historical context. Mirror symmetry, which has been a major area of mathematical research since the early 1990s, originated in string theory, which itself originated in the late 1960s as a candidate mathematical framework for understanding the scattering of hadrons. We will deal very little with string theory itself, but it will be instructive to note that scattering in string theory is understood not through the interactions of 1-dimensional trajectories of 0-dimensional particles, but instead through 2-dimensional worldsheets that describe the trajectories, merging, and splitting of 1-dimensional strings (hence the name). Following the development of quantum chromodynamics and the acceptance of the standard model in the mid 1970s, the focus of theoretical high energy physics largely shifted to studying "quantum gravity," i.e. trying to reconcile gravity with the standard model, and this is the context in which string theory is most well known.

String theory is perhaps better understood as a family of theories, rather than a single theory, and much of the string theoretic search for a theory of quantum gravity can be framed as a search for a specific string theory that reproduces the standard model. To give a simplified description of this process, we first note the following: string theories work poorly in 4-dimensional spacetime, with the original string theories working only in 26 dimensions and the later supersymmetric string theories working only in 10 dimensions. The extra dimensions are understood to be small in some appropriate senses (e.g. metrically small and topologically compact), and their structure determines much of the physics of the corresponding string theory. In the supersymmetric context, these 10-dimensional spacetimes can be understood as being our usual 4-dimensional spacetime plus an additional 6 dimensions, where the extra 6 dimensions have the structure of a Calabi-Yau manifold, discussed further below. Different choices of Calabi-Yau manifolds can be interpreted as different choices of a specific string theory, and much of the string theoretic search for quantum gravity can be interpreted as looking for a Calabi-Yau manifold that would produce the physics of the standard model.

In the 1980s, people began finding pairs of Calabi-Yau manifolds where both produced the same physics. This is the origin of mirror symmetry. The topic attracted significant interest from mathematicians beginning in the early 1990s, following the seminal paper of Candelas, de la Ossa, Green, and Parkes [2], which used mirror symmetry to calculate a quantity in enumerative algebraic geometry, namely the number of rational curves in a quintic 3-fold, that had long eluded mathematicians. Mirror symmetry has been a major area of study in mathematics ever since, taking a wide range of forms and expanding well past the original physical context. For instance, the primary setting for this note is log Calabi-Yau surfaces, which are 4-dimensional instead of 6-dimensional, and which are non-compact.

Mirror symmetry: the A-Side and the B-Side

Calabi-Yau manifolds are highly structured spaces. The two structures we will most focus on are the symplectic structure and the complex structure. One can think of the symplectic structure as a means of assigning an area or energy to a 2-dimensional subspace, such as the worldsheet of a string; and one can think of the complex structure as providing a natural right-angle rotation at every point, which must send any vector tangent to a worldsheet to another vector tangent to the worldsheet, and hence constraining the set of possible worldsheets. Mirror symmetry can be thought of as a symmetry between the symplectic structure of one Calabi-Yau manifold and the complex structure of another Calabi-Yau manifold, and each is called the mirror of the other. The symplectic and complex structure sides of this symmetry are often referred to as the "A-side" and "B-side," respectively.

There are multiple approaches to making the loose idea given above rigorous. We will be focusing on one known as the Strominger-Yau-Zaslow Conjecture. The most basic form of the SYZ conjecture posits that a Calabi-Yau manifold on the A-side can be expressed as a special Lagrangian fibration, and that the B-side mirror manifold will then be given by the dual fibration. In the string theory terminology, this is T-duality. Loosely speaking, this amounts to expressing the A-side as a union of well-behaved tori (called the fibers of the fibration) that behave nicely with respect to the symplectic structure, and then using this expression to create another family of well-behaved tori (the dual fibration) which have an induced complex structure. This new union of tori is then the B-side. The fibers and base of the fibration are each half the dimension of the Calabi-Yau manifold, e.g. 3 and 3 in the case of the Calabi-Yau manifolds appearing in string theory. The simplest example is a compact 2-dimensional Calabi-Yau manifold, which must be topologically a torus, which can be expressed as a union of 1-dimensional tori in a 1-dimensional family, i.e. a union of circles in a circular family.

Unfortunately, the simple form of the SYZ conjecture is false in general, in that many Calabi-Yau manifolds cannot actually be expressed as a special Lagrangian fibration. However, many of them can still be expressed as a singular special Lagrangian fibration, in which a positive codimensional family of the fibers can fail to be smooth. The base of the fibration will then also have singularities, corresponding to the singular fibers. The dual fibration will not be mirror to the original A-side Calabi-Yau, but it can often be altered, using what are called instanton corrections, to produce an appropriate mirror.

Intuitively, these corrections arise from the presence of objects analogous to the world-sheets of strings appearing in string theory. They are 2-dimensional surfaces inside the Calabi-Yau manifold known as holomorphic curves. In the case we will be considering, these will be discs, where the 1-dimensional boundary of the disc is required to lie on one of the fibers of the (singular) special Lagrangian fibration. It is often straightforward to find some of the holomorphic discs, but finding all of them can be difficult. In the case we will consider, there will be a finite number of families of basic discs, which admit a straightforward description, and all other discs will appear from interactions between basic discs in different families. In sufficiently nice situations, these families of discs have their boundaries lying on corresponding families of Lagrangians in the fibration, forming codimension 1 sets called walls in the base of the fibration. Our A-side scattering diagrams will then consist of these walls, together with wall-crossing functions encoding further information about how the discs interact, or "scatter."

A-Side scattering diagrams for log Calabi-Yau surfaces

We turn now to constructing scattering diagrams, and our discussion will be considerably more technical from this point on. In our setting, a log Calabi-Yau surface Y is a 4-dimensional Kähler manifold (called a surface because it is 2-dimensional as a complex manifold) obtained from a compact rational surface X with a meromorphic volume form Ω with simple poles along a divisor D. We obtain Y by taking $X \setminus D$. They are called log Calabi-Yau because Ω restricts to a holomorphic volume form on Y and because Ω has log singularities along D. We will consider in particular those log Calabi-Yau surfaces arising from Looijenga pairs, i.e. those pairs (X, D) where D has at least one singular point. The techniques we discuss will only apply in this case, and mirror symmetry in the case where D is smooth was handled earlier by Lau-Lee-Lin [10].

By first performing iterated blowups at singular points of D, we can assume that Y arises as $\widetilde{X}\backslash\widetilde{D}$ for a pair $(\widetilde{X}, \widetilde{D})$, where $\widetilde{X}$ is obtained from a toric surface $\overline{X}$ with toric boundary $\overline{D}$ by iterated blow up at points on $\overline{D}$ and its proper transforms. The divisor $\widetilde{D}$ is then the final proper transform of $\overline{D}$. This blowup $\pi : (\widetilde{X}, \widetilde{D}) \to (\overline{X}, \overline{D})$ is called a toric model for Y, see Gross-Hacking-Keel [7]. If this $\widetilde{X}$ is actually toric itself, i.e. $\widetilde{X} = \overline{X}$, then Y is precisely $(\mathbb{C}^*)^2$ and the toric moment map fibration is a smooth special Lagrangian fibration. Otherwise, we need to construct a singular special Lagrangian fibration. We use the local S^1 action around each non-toric blowup to construct a symplectic form on Y that coincides with the pullback by π of the symplectic form on $\widetilde{X}$ when sufficiently far away from the exceptional locus. We then follow Gross [6] and use the reduction by this local S^1 action to produce a local singular special Lagrangian fibration around each exceptional divisor. This singular fibration has singularities precisely at the fixed points of the S^1 action, and hence has precisely one singular fiber for each non-toric blowup used to construct $\widetilde{X}$ from $\overline{X}$. Each of these singular fibers is an immersed S^2. These local fibrations then patch to a global special Lagriangian fibration. The base of this fibration is an integral affine manifold with singularities corresponding to the singular fibers. Sufficiently far away from the blowup locus, these coincide with the pullback by π of the standard moment fibers in $\widetilde{X} \setminus \widetilde{D}$. We refer to these particular fibers as admissible fibers.

We now consider the holomorphic discs in Y with boundary on an admissible fiber L. The holomorphic discs in $\overline{X}$ with boundary on $\pi(L)$ were completely classified by Cho-Oh [3], and we hence have a corresponding classification of the holomorphic discs in Y with boundary on L. In particular, all such discs in Y are proper transforms of discs in $\overline{X}$ that intersect the boundary $\overline{D}$ only at the points that get blown up by π (and only to appropriate orders). For generic choice of blowup points in $\overline{D}$ there are no closed holomorphic curves in Y. Combining this fact with Cho-Oh [3], we see that every holomorphic disc in Y with boundary on L is regular, in the sense that the linearization of the $\overline{\partial}$ map is surjective. Since each such disc furthermore has Maslov index 0, a dimension argument shows the locus in the base of the fibration corresponding to admissible fibers that bound holomorphic discs is a countable union of 1-dimensional submanifolds. These are the walls of our scattering diagram. The wall-crossing functions are defined Floer theoretically: given two admissible fibers L_0 and L_1 on either side of a wall and a short path between them, the work of Fukaya-Oh-Ohta-Ono [5] and Fukaya [4] gives a pseudoisotopy of A_∞-algebras between the A_∞-algebra structures on $H^*(L_0)$ and $H^*(L_1)$. This pseudoisotopy is independent of the choice of path, up to a pseudoisotopy of pseudoisotopies, and we can use it to define a wall-crossing function from $H^1(L_0)$ to $H^1(L_1)$. The path independence furthermore implies that, if one takes a composition of wall-crossing functions obtained by following a closed loop, the composition must be the identity map. This is referred to as the consistency of the scattering diagram.

In the case of walls coming from the proper transform of Maslov index 2 discs in $\overline{X}$, the wall-crossing function can be computed explicitly using, for instance, techniques of Lin [11]. We refer to these as initial walls. Roughly speaking, these walls emerge from the zones in the base of the fibration near the singular fibers, where we have limited control, and we require some technical energy arguments to disregard the possibility of walls other than these initial walls coming out of these inadmissible zones. Modulo this issue, work of Kontsevich and Soibelman [9] then shows that all of the other walls and wall-crossing functions are uniquely determined by the initial walls. In some sense, this gives a complete description of our scattering diagram and the instanton corrections necessary to construct the mirror to our log Calabi-Yau surface, but more clarity is needed to actually use this description. This brings us to tropical geometry.

Tropical geometry and agreement with B-side scattering diagrams

The walls we described above are in general quite tricky to understand, especially outside of the admissible regions of the base of the fibration. However, it turns out that we can take a certain limit, corresponding to modifying the complex structure on Y and moving toward the large complex structure limit, and arrive at a scattering diagram where the walls are simply rays in $\mathbb{R}^2$. This is an example of tropicalization, and the result is a tropical scattering diagram. One can think of this process as "straightening out" the walls of the scattering diagram so that they are easier to understand. The resulting tropical diagram is completely combinatorial in nature.

This is not the first appearance of tropical scattering diagrams in the literature, and in [1] we show that our scattering diagram coincides with a corresponding scattering diagram developed by Gross-Pandharipande-Siebert [8] using B-side information about the pair $(\widetilde{X}, \widetilde{D})$, namely counts of A^1 curves. This confirms a folklore understanding of the walls appearing in the GPS scattering diagram as holomorphic discs.

In addition to their applications in understand the log Calabi-Yau surface Y, these scattering diagrams are useful for understanding Fano compactifications of such surfaces.

We close with a figure demonstrating this hands-on usefulness. By work of Pascaleff-Tonkonog [12] and Venugopalan-Woodward [13], it is known that the degree 3 del Pezzo surface, i.e. the blowup of $\mathbb{CP}^2$ at 6 generic points, contains a Lagrangian that bounds 21 Maslov index 2 discs. With our tropical scattering diagram, we can directly visualize the associated discs. See the caption for a detailed description of the figure.

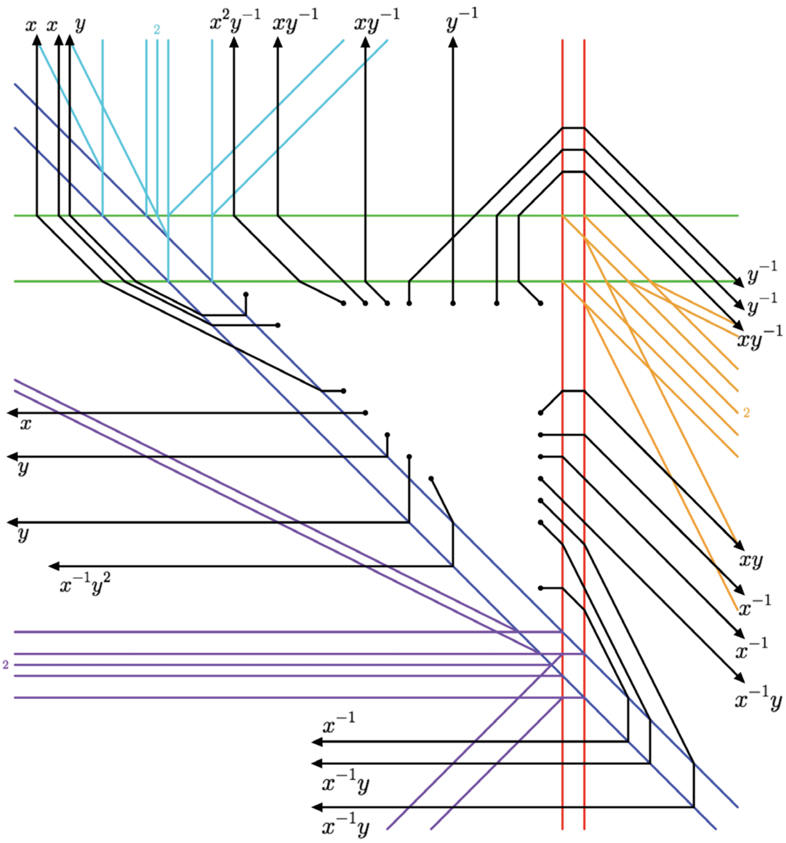

Figure 1: Tropical picture of 21 discs in the degree 3 del Pezzo surface. Each point in the picture corresponds to a Lagrangian torus. The black broken lines in the diagram correspond to the Maslov index 2 discs, and the lines and rays of other colors are part of the scattering diagram. The full scattering diagram contains infinitely many rays, but all relevant rays are pictured. Note how the walls scatter off one another, producing new walls when they intersect.

The allowable angles for the infinite part of each broken line are determined by the toric fan for $\mathbb{CP}^2$, and the rules for determining how the broken lines are allowed to bend are determined by the wall-crossing functions (not pictured). Any point in the central triangular region of the scattering diagram corresponds to a Lagrangian that bounds all 21 of the pictured discs, though for legibility they are each drawn coming from a separate point. The polynomials next to each broken line indicate the term that the corresponding disc contributes to the potential function of the Lagrangian.

References

[1] S. Bardwell-Evans, M.-W. M. Cheung, H. Hong, and Y.-S. Lin, *Scattering diagrams from holomorphic discs in log Calabi-Yau surfaces*, Journal of Differential Geometry **130** (2025), 71–150.

[2] P. Candelas, X. C. de la Ossa, P. S. Green, and L. Parkes, *A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory*, Nuclear Phys. B **359** (1991), no. 1, 21–74.

[3] C.-H. Cho and Y.-G. Oh, *Floer cohomology and disc instantons of Lagrangian torus fibers in Fano toric manifolds*, Asian J. Math. **10** (2006), no. 4, 773–814.

[4] K. Fukaya, *Cyclic symmetry and adic convergence in Lagrangian Floer theory*, Kyoto J. Math. **50** (2010), no. 3, 521–590.

[5] K. Fukaya, Y.-G. Oh, H. Ohta, and K. Ono, *Lagrangian intersection Floer theory: anomaly and obstruction. Parts I and II*, AMS/IP Studies in Advanced Mathematics, vol. 46, American Mathematical Society, Providence, RI, 2009.

[6] M. Gross, *Examples of special Lagrangian fibrations*, Symplectic geometry and mirror symmetry (Seoul, 2000), 2001, pp. 81–109.

[7] M. Gross, P. Hacking, and S. Keel, *Mirror symmetry for log Calabi-Yau surfaces I*, Publ. Math. Inst. Hautes Études Sci. **122** (2015), 65–168.

[8] M. Gross, R. Pandharipande, and B. Siebert, *The tropical vertex*, Duke Math. J. **153** (2010), no. 2, 297–362.

[9] M. Kontsevich and Y. Soibelman, *Affine structures and non-Archimedean analytic spaces*, The unity of mathematics, 2006, pp. 321–385.

[10] S.-C. Lau, T.-J. Lee, and Y.-S. Lin, *On the complex affine structures of SYZ fibration on del Pezzo surfaces*, preprint, arXiv: 2005.04825.

[11] Y.-S. Lin, *Correspondence Theorem between Holomorphic Discs and Tropical Discs on K3 Surfaces*, J. Differ. Geom. **117** (2021), no. 1, 41–92.

[12] J. Pascaleff and D. Tonkonog, *The wall-crossing formula and Lagrangian mutations*, Adv. Math. **361** (2020), 106850, 67.

[13] S. Venugopalan and C. Woodward, *Tropical Fukaya algebra*, preprint 2020, arXiv: 2004.14314.

Simply connected positive Sasakian 5-manifolds

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Introduction

A Riemannian manifold (M, g) is called Sasakian if the cone metric $r^2 g + dr^2$ defines a Kähler metric on $M \times \mathbb{R}^+$. Let J be a complex structure on the Kähler manifold $M \times \mathbb{R}^+$, then the vector field $J\left(r\frac{\partial}{\partial r}\right)$ on $M \times \mathbb{R}^+$ induces a Reeb vector field on M. If the orbits of the Reeb vector field is closed, then the Sasakian structure is called quasi-regular.

By Rukimbira's results ([Ruk95, Theorem 6]), every compact Sasakian manifold M can accommodate a quasi-regular Sasakian structure. Note that the orbits are circles, but the S^1-action need not be free. Thus, each compact Sasakian manifold can be written as the unit circle subbundle of a holomorphic Seifert $\mathbb{C}^*$-bundle over a cyclic complex algebraic orbifold (X, Δ), where the boundary divisor $\Delta = \sum\left(1 - \frac{1}{m_i}\right) R_i$ describes the branch divisors R_i's of the orbifold with their multiplicities m_i's. If the orbifold (X, Δ) is Fano, i.e., its first orbifold Chern class $c_1^{orb}(X, \Delta) = -K_X - \Delta$ is positive, then the Sasakian structure on M is called positive.

Conversely, the Seifert S^1-bundle can be characterized as follows:

Theorem 1 ([Kol05]). *There is a one-to-one correspondence between Seifert S^1-bundles over (X, Δ) and the following data:*

(1) For each prime divisor R_i an integer $0 \le b_i < m_i$, relatively prime to m_i,

(2) a linear equivalence class of Weil divisors B on X.

If the first Chern class $c_1(M/X) = B + \sum \frac{b_i}{m_i} R_i$ of the Seifert S^1-bundle $M \to (X, \Delta)$ is ample, then M admits a Sasakian structure ([BG08, Theorem 7.5.2 and Proposition 7.5.23]).

Furthermore, we are also interested in the existence of the Einstein metric on positive Sasakian manifolds. The metric g on M is called Einstein if $\mathrm{Ric} = \lambda g$ for some $\lambda \in \mathbb{R}$. If $c_1(M/X)$ is proportional to $c_1^{orb}(X, \Delta)$, then (X, Δ) admits an orbifold Kähler-Einstein metric if and only if M admits a Sasaki-Einstein metric [Kol05, Theorem 1.1].

The following questions then naturally arise:

Question. *Which manifolds admit a positive Sasakian structure? If so, do they also allow an Einstein metrics?*

Simply connected 5-manifolds

Our goal is to answer these question about closed simply connected 5-manifolds. To do this end, we recall their classification.

First, closed simply connected spin 5-manifolds can be described in [Sma62] as follows: For a positive integer m, up to diffeomorphisms, there is a unique closed simply connected spin 5-manifold M_m with $\mathrm{H}_2(M_m, \mathbb{Z}) = (\mathbb{Z}/m\mathbb{Z})^{\oplus 2}$. Furthermore, a closed simply connected spin 5-manifold M is of the form

$$M = kM_\infty \# M_{m_1} \# \dots \# M_{m_r},$$

where kM_∞ is the k-fold connected sum of $S^2 \times S^3$ for a non-negative integer k and m_i is a positive integer greater than 1 with m_i dividing m_{i+1} ([Sma62, Theorem A]).

Meanwhile, closed simply connected non-spin 5-manifolds are described in [Bar65]. For instance, the non-trivial S^3-bundle over S^2, denoted by X_∞, is a closed simply connected non-spin 5-manifold. Every closed simply connected non-spin 5-manifold with trivial third integral Stiefel-Whitney class is of the form

$$X_\infty \# M,$$

where M is a closed simply connected spin 5-manifold ([Bar65, Lemma 1.2 and Theorem 2.3]).

Known results

Kollár identified the possible candidates that can admit a positive Sasakian structure by analyzing cyclic del Pezzo orbifolds:

Theorem 2 ([Kol05], [Kol09]). *Let M be a closed simply connected 5-manifold that admits a positive Sasakian structure.*

(1) The torsion part of $\mathrm{H}_2(M, \mathbb{Z})$ is one of the following:

$$(\mathbb{Z}/m\mathbb{Z})^{\oplus 2},\ (\mathbb{Z}/5\mathbb{Z})^{\oplus 4},\ (\mathbb{Z}/4\mathbb{Z})^{\oplus 4},$$
$$(\mathbb{Z}/3\mathbb{Z})^{\oplus 4},\ (\mathbb{Z}/3\mathbb{Z})^{\oplus 6},\ (\mathbb{Z}/3\mathbb{Z})^{\oplus 8}, (\mathbb{Z}/2\mathbb{Z})^{\oplus 2n},$$

where $m, n \geq 1$.

(2) If $\mathrm{H}_2(M, \mathbb{Z}) = (\mathbb{Z}/m\mathbb{Z})^{\oplus 2}$ for $m \geq 1$, then m is not divisible by 30.

(3) If $\mathrm{H}_2(M,\mathbb{Z})_{\mathrm{tor}} = (\mathbb{Z}/5\mathbb{Z})^{\oplus 4}$ *or* $(\mathbb{Z}/3\mathbb{Z})^{\oplus 8}$*, then* $\operatorname{rank}(\mathrm{H}_2(M,\mathbb{Z})) = 0$.

(4) If $\mathrm{H}_2(M,\mathbb{Z})_{\mathrm{tor}} = (\mathbb{Z}/4\mathbb{Z})^{\oplus 4}$*, then* $\operatorname{rank}(\mathrm{H}_2(M,\mathbb{Z})) \leq 1$.

(5) If $\mathrm{H}_2(M,\mathbb{Z})_{\mathrm{tor}} = (\mathbb{Z}/3\mathbb{Z})^{\oplus 6}$*, then* $\operatorname{rank}(\mathrm{H}_2(M,\mathbb{Z})) \leq 1$.

(6) If $\mathrm{H}_2(M,\mathbb{Z})_{\mathrm{tor}} = (\mathbb{Z}/m\mathbb{Z})^{\oplus 2}$ *for* $m \geq 12$*, then* $\operatorname{rank}(\mathrm{H}_2(M,\mathbb{Z})) \leq 8$.

Among these, the following cases have been established in previous works:

Theorem 3 ([BG06], [BN10], [JP23], [Kol05], [Kol07], [Kol09], [PW21])**.** *The following closed simply connected spin 5-manifolds admit positive Sasaki-Einstein structures:*

(1) $M_m, nM_2, 2M_3, 3M_3, 4M_3, 2M_4, 2M_5$*;*

(2) sM_∞ *for* $s \geq 1$*;*

(3) $(k+1)M_\infty \# M_m$ *for* $0 \leq k \leq 7$*,* $m \geq 2$*.*

(4) $M_\infty \# 2M_4$*,* $M_\infty \# 3M_3$*,* $M_\infty \# 2M_3$*.*

Main results and remaining cases

First, we construct the non-spin counterparts in Theorem 3 which allow positive Sasakian structures by varying the first Chern class $c_1(M/X)$:

Theorem 4 ([JPW25])**.** *The following closed simply connected 5-manifolds admit positive Sasakian structures:*

(1) $X_\infty \# sM_\infty$ *for* $s \geq 0$*;*

(2) $X_\infty \# kM_\infty \# M_m$ *for* $0 \leq k \leq 7$*,* $m \geq 2$*;*

(3) $X_\infty \# 2M_4$*,* $X_\infty \# 3M_3$*,* $X_\infty \# 2M_3$*,* $X_\infty \# M_\infty \# 3M_3$*;*

Since any simply connected Sasaki-Einstein manifold is spin, these manifolds cannot admit Einstein metrics.

On the other hand, we also construct new closed simply connected positive Sasakian 5-manifolds:

Theorem 5 ([JPW25])**.** *(1)* $2M_3 \# 2M_3$ *admits a positive Sasaki-Einstein structure;*

(2) $rM_\infty \# nM_2$ *and* $X_\infty \# rM_\infty \# nM_2$ *admit positive Sasakian structures for* $r, n \geq 0$.

Unfortunately, the Sasakian structures on $rM_\infty \# nM_2$ that we construct do not admit Einstein metrics for $r, n \geq 0$.

The construction of such 5-manifolds is outlined as follows:

(1) Find an appropriate del Pezzo orbifold (S, Δ) and an ample divisor $c_1(M/S)$.

(2) Verify the topological properties of M: spin-ness, smoothness, simply connectedness.

(3) If M is a simply connected manifold, compute $H_2(M, \mathbb{Z})$.

(4) If, in addition, $c_1(M/S)$ is proportional to $c_1(S, \Delta)$, then check whether (S, Δ) admits an orbifold Kähler-Einstein metric or not.

For instance, consider the case of $(k+1)M_\infty \# M_n$ and $X_\infty \# kM_\infty \# M_m$ for $2 \leq k \leq 7$ and $m \geq 2$: Let S_k be a blowup of $\mathbb{P}^2$ at general $k+1$ points and D_k be its smooth anticanonical curve. Then, $\left(S_k, \left(1 - \frac{1}{m}\right) D_k\right)$ is a del Pezzo orbifold. Now, let $M^0_{k,m}$ and $M^1_{k,m}$ be Seifert S^1-bundles over $\left(S_k, \left(1 - \frac{1}{m}\right) D_k\right)$ corresponding to the ample Chern classes

$$c_1(M^0_{k,m}/S_k) = \frac{1}{m} D_k, \quad c_1(M^1_{k,m}) = H_k + \frac{1}{m} D_k,$$

respectively. Here, H_k is a pullback of a general line in $\mathbb{P}^2$.

Then, one can see that $M^i_{k,m}$ is a simply connected manifold with

$$H_2(M^i_{k,m}, \mathbb{Z}) = \mathbb{Z}^{k+1} \oplus (\mathbb{Z}/m\mathbb{Z})^{\oplus 2},$$

for $i = 0, 1$. Furthermore, $M^0_{k,m}$ is a spin manifold, but $M^1_{k,m}$ is not. This shows that $(k+1)M_\infty \# M_m$ and $X_\infty \# kM_\infty \# M_m$ admit positive Sasakian structures.

Moreover, one can check that $\left(S_k, \left(1 - \frac{1}{m}\right) D_k\right)$ admits an orbifold Einstein metric by verifying its K-stability. Since

$$c_1(M^0_{k,m}/S_k) = c_1^{orb} \left(S_k, \left(1 - \frac{1}{m}\right) D_k\right),$$

this concludes that $(k+1)M_\infty \# M_m$ also allows an Einstein structure.

To complete the picture, it remains to understand the following two cases:

Conjecture 6. *Let M be a closed simply connected 5-manifold that allows a positive Sasakian structure.*

(1) If $\mathrm{H}_2(M, \mathbb{Z})_{\mathrm{tor}} = (\mathbb{Z}/3\mathbb{Z})^{\oplus 4}$*, then* $\mathrm{rank}(\mathrm{H}_2(M, \mathbb{Z})) \leq 2$.

(2) If $\mathrm{rank}(\mathrm{H}_2(M, \mathbb{Z})) \geq 9$*, then*

$$\mathrm{H}_2(M, \mathbb{Z})_{\mathrm{tor}} = (\mathbb{Z}/2\mathbb{Z})^{\oplus 2n}$$

for some non-negative integer n. If in addition M admits a Sasaki-Einstein structure, then the torsion is trivial, i.e., $n = 0$.

References

[Bar65] D. Barden. Simply connected five-manifolds. *Ann. of Math. (2)*, 82:365–385, 1965.

[BG06] Charles P. Boyer and Krzysztof Galicki. Einstein metrics on rational homology spheres. *J. Differential Geom.*, 74(3):353–362, 2006.

[BG08] Charles P. Boyer and Krzysztof Galicki. *Sasakian geometry*. Oxford Mathematical Monographs. Oxford University Press, Oxford, 2008.

[BN10] Charles P. Boyer and Michael Nakamaye. On Sasaki-Einstein manifolds in dimension five. *Geom. Dedicata*, 144:141–156, 2010.

[JP23] Dasol Jeong and Jihun Park. Simply connected Sasaki-Einstein 5-manifolds: old and new. In *Birational geometry, Kähler-Einstein metrics and degenerations*, volume 409 of *Springer Proc. Math. Stat.*, pages 665–680. Springer, Cham, [2023] ©2023.

[JPW25] Dasol Jeong, Jihun Park, and Joonyeong Won. Simply connected positive Sasakian 5-manifolds. *J. Lond. Math. Soc. (2)*, 112(1):Paper No. e70227, 28, 2025.

[Kol05] János Kollár. Einstein metrics on five-dimensional Seifert bundles. *J. Geom. Anal.*, 15(3):445–476, 2005.

[Kol07] János Kollár. Einstein metrics on connected sums of $S^2 \times S^3$. *J. Differential Geom.*, 75(2):259–272, 2007.

[Kol09] János Kollár. Positive Sasakian structures on 5-manifolds. In *Riemannian topology and geometric structures on manifolds*, volume 271 of *Progr. Math.*, pages 93–117. Birkhäuser Boston, Boston, MA, 2009.

[PW21] Jihun Park and Joonyeong Won. Simply connected Sasaki-Einstein rational homology 5-spheres. *Duke Math. J.*, 170(6):1085–1112, 2021.

[Ruk95] Philippe Rukimbira. Chern-Hamilton's conjecture and K-contactness. *Houston J. Math.*, 21(4):709–718, 1995.

[Sma62] Stephen Smale. On the structure of 5-manifolds. *Ann. of Math. (2)*, 75:38–46, 1962.

Fukaya Categories of Hyperplane Arrangements

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Introduction

The Fukaya category of a symplectic manifold is an A_∞-category whose objects are Lagrangian submanifolds and whose morphism spaces are given by Floer chain complexes. Its construction and computation for broad classes of symplectic manifolds constitute a central problem in symplectic geometry. In [6], we compute the Fukaya category associated to a class of symplectic manifolds arising from hyperplane arrangements and confirm the conjecture of Licata–Lauda–Manin [4].

Wrapped Fukaya categories

Definition 1. Let (X, ω) be an exact symplectic manifold. We say (X, ω) is a **Liouville manifold** if there exists a complete vector field Z dual to a primitive of ω.

Topologically, a Liouville manifold X has cylindrical infinity, meaning that there exists a compact domain $X_0 \subset X$ such that $X = X_0 \cup_{\partial X_0} \partial X_0 \times [1, +\infty)$, where the Liouville structure on $\partial X_0 \times [1, +\infty)_r$ is $(\omega = d(r\lambda|_{\partial X_0}), Z = r\partial_r)$. Recall that a **Lagrangian submanifold** of X is a submanifold $L \subseteq X$ of half dimension such that $\omega|_L = 0$.

Definition 2. A Lagrangian submanifold $L \subseteq X$ is **exact** if the restrction of a primitive of $\omega|_L$ is exact on L. It is **cylindrical** if Z is tangent to L near infinity.

To any two transversely intersecting exact cylindrical Lagrangian submanifolds $L, K \subseteq X$, we can associate a $(\mathbb{Z}/2)$-graded free $\mathbb{Z}$-module called the Floer cochain complex

$$CF^*(L, K) := \bigoplus_{p \in L \cap K} \mathfrak{o}_p,$$

where $\mathfrak{o}_p$ is the orientation line associated to $p \in L \cap K$. The complex $CF^*(L, K)$ admits a differential defined by an oriented count of holomorphic strips with boundary on $L \cap K$, whose cohomology gives the Lagrangian Floer homology group $HF^*(L, K)$.

Definition 3. An isotopy $\{L_t\}$ of exact cylindrical Lagrangian submanifolds $L_t \subseteq X$ is called **positive** if for some, equivalently any, contact form α on $\partial_\infty X$, we have $\alpha(\partial_t \partial_\infty L_t) > 0$.

A positive isotopy $(L_0 \rightsquigarrow L_1) := \{L_t\}_{0 \le t \le 1}$ of Lagrangian submanifolds such that L_0, L_1 are transversal to K induces a chain map

$$CF^*(L_0, K) \to CF^*(L_1, K)$$

between Floer cochain complexes. We define the **wrapped Floer cohomology** associated to L and K by the homotopy colimit

$$HW^*(L, K) := \varinjlim_{L \rightsquigarrow L'} HF^*(L', K).$$

The **wrapped Fukaya category** of X, denoted by $\mathcal{W}(X)$, is an A_∞-category whose objects are exact cylindrical Lagrangian submanifolds $L \subseteq X$ and the morphism space between L and K is the localization of $CF^*(L, K)$ along continuation elements. Furthermore, one can decorate X by choosing a cylindrical submanifold $S \subset \partial_\infty X$ which prevents wrapping through it. This is called a **stop**, and the pair (X, S) - or simply denoted by X - is called a **Liouville sector** if, near infinity, S is locally isomorphic to a product $S_0 \times \mathbb{C}$ for some compact domain S_0 (see [3, Section 1] for the precise definitions). In particular, when (X, ω, Z) is Weinstein, meaning that there exists an exhausting Morse function $\phi : X \to \mathbb{R}$ which is Lyapunov for Z, Ganatra–Pardon–Shende developed sheaf-theoretic techniques to compute the wrapped Fukaya category, the so-called **sectorial descent** [3].

Theorem 4 (Sectorial Descent [3], Theorem 1.35)**.** *For any Weinstein sectorial covering $X_1, \cdots, X_k$ of a Weinstein sector X, the induced functor*

$$\operatorname{hocolim}_{\emptyset \ne I \subseteq \{1, \cdots, k\}} \mathcal{W}\left(\bigcap_{i \in I} X_i\right) \xrightarrow{\simeq} \mathcal{W}(X)$$

is a pre-triangulated equivalence.

Polarized hyperplane arrangements

We introduce a certain Weinstein sector from combinatorial data of hyperplane arrangements.

A **polarized hyperplane arrangement** consists of a triple $\mathbb{V} = (V, \eta, \xi)$, where $V \subset \mathbb{R}^n$ is a d-dimensional subspace, $\eta \in \mathbb{R}^n/V$ determines n affine hyperplanes in $\mathbb{R}^d$, and $\xi \in (\mathbb{R}^n)^*/V^\perp$ is a polarization. We say $\mathbb{V}$ is **simple** if the non-empty intersection of k hyperplanes is codimension k for $k \le n$.

The real hyperplanes divide $\mathbb{R}^d$ into chambers indexed by sign sequences $\alpha \in \{+, -\}^n$. A chamber is called **feasible** if it is non-empty, and **bounded** if the linear functional ξ is bounded above on it. Let $P(\mathbb{V})$ denote the set of bounded-feasible chambers. Complexifying the arrangement gives hyperplanes $H_i \subset \mathbb{C}^d$, and one defines

$$M(\mathbb{V}) := \mathbb{C}^d \setminus \bigcup_{i=1}^{n} H_i.$$

This is a very affine variety, which embeds into $(\mathbb{C}^*)^n$, where H_i maps to the i-th coordinate hyperplane of $\mathbb{C}^n$. Note that the standard Stein structure on $(\mathbb{C}^*)^n$ induces a Weinstein structure on $M(\mathbb{V})$. We also take a generic hypersurface in $M(\mathbb{V})$ defined by the polarization ξ, abusively denoted again by ξ. Although ξ is not sectorial on the nose, one can deform the Weinstein structure to make it sectorial (see [6, Proposition 3.14]). To each bounded feasible chamber $\alpha \in P(\mathbb{V})$, one associates a Lagrangian submanifold $L_\alpha \subset M(\mathbb{V})$, given by the corresponding real chamber inside the real locus $M(\mathbb{V}) \cap \mathbb{R}^d$. These L_α are exact, non-compact, and cylindrical at infinity.

Example 5. Let V be the image of the linear map $A : \mathbb{R}^2 \to \mathbb{R}^3$ given by $A(x, y) = (-x, -y, -x - y)$. Let $\eta = (0, 0, -1)$ and ξ be represented by $(-1, 1, 0)$. Then we have the following arrangement of hyperplanes in $\mathbb{R}^2$ with bounded-feasible chambers shaded. We consider the following hyperplane arragements in $\mathbb{R}^2$.

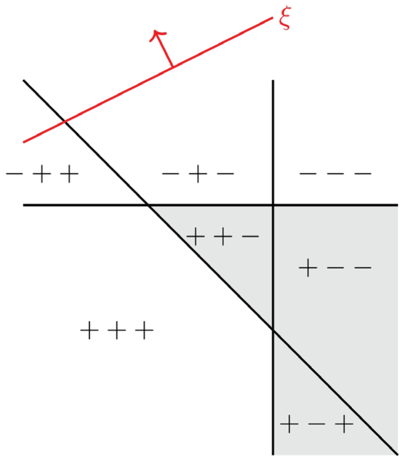

Figure 1: Example 5

Main results

Let $\mathbb{V}$ be a simple polarized hyperplane arrangement. The following results were established in the case when $\mathbb{V}$ is cyclic by Licata–Lauda–Manin [4]. The novelty here is the extension to the case of simple hyperplane arrangements.

Theorem 6 (Generation)**.** *[6] The partially wrapped Fukaya category $\mathcal{W}(M(\mathbb{V}), \xi)$ is generated by the Lagrangian submanifolds $\{L_\alpha\}_{\alpha \in P(\mathbb{V})}$.*

Define the Fukaya A_∞-algebra

$$\widetilde{B}_{\mathbb{V}} = \bigoplus_{\alpha, \beta \in P(V)} CF^*(L_\alpha, L_\beta),$$

Theorem 7 (Formality)**.** *[6] There is a quasi-isomorphism of A_∞-algebras*

$$\widetilde{B}_{\mathbb{V}} \simeq \widetilde{B}(\mathbb{V}).$$

where $\widetilde{B}(\mathbb{V})$ is a a graded deformation of the hypertoric convolution algebra introduced in [2]. In particular, $\widetilde{B}_{\mathbb{V}}$ is formal.

Remark 8. Associated to the same hyperplane arrangement $\mathbb{V}$ is a hypertoric variety $M_{\mathbb{V}}$, a hyperkähler quotient with a Hamiltonian T^d-action. Lekili–Segal conjectured that the equivariant wrapped Fukaya category of $M_{\mathbb{V}}$ is equivalent to the wrapped Fukaya category of $M(\mathbb{V})$ [5]. The above results provides a strong evidence of their conjecture in the partially wrapped setting.

Idea of the proof

The proof relies on **sectorial descent** (Theorem 4). For simplicity, we present the two-dimensional case of Theorem 6. Note that the inductive argument extends to higher dimensions, and Theorem 7 then follows from standard techniques (cf. [1]).

Consider the arrangement of hyperplanes in $\mathbb{R}^2$ depicted in Figure 1. One can further cut $M(\mathbb{V})$ into 5 pieces as in Figure 2.

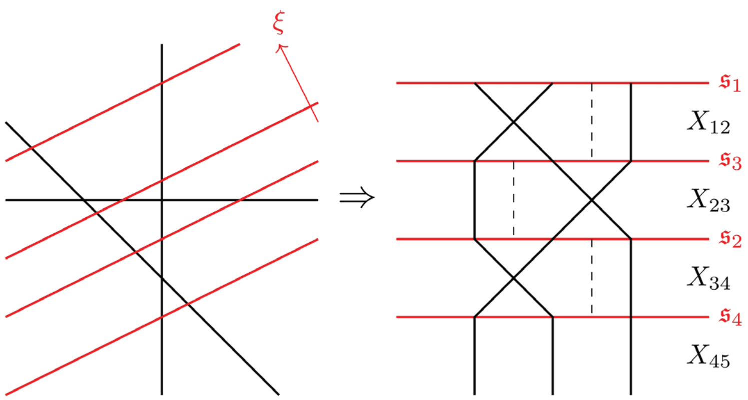

Figure 2: Decomposing 2-dimensional pair-of-pants into sectors

For each of these subsectors, we perform a further cut (depicted as dashed lines in Figure 2) by hyperplanes which are transverse to the hyperplane defined by the polarization ξ, cutting each sector into "cubes" as shown in Figure 2. Each cube is either isomorphic to $(\mathbb{C}^*)^2$ with four stops, or to the stabilization of the 2-dimensional pair-of-pants $\mathbb{C} \setminus \{z_1, z_2, z_3\}$. It is well known that the wrapped Fukaya category of $(\mathbb{C}^*)^2$ is generated by one of the four real quadrants (a cotangent fiber), and adding stops introduces linking disks associated to these stops as new generators.

Then we perform several surgeries among those Lagrangians and linking disks to show that, under the sectorial descent (Theorem 4), the set of chamber Lagrangians $\{L_\alpha \mid \alpha \in P(\mathbb{V})\}$ generates $\mathcal{W}(X)$.

References

[1] D. Auroux. *Speculations on homological mirror symmetry for hypersurfaces in* $(\mathbb{C}^*)^n$ Surveys in Differential Geometry, 22:1–47, 2017.

[2] T. Braden, A. Licata, N. Proudfoot, and B. Webster, *Gale duality and Koszul duality* Adv. Math., 225(4):2002–2049, 2010.

[3] S. Ganatra, J. Pardon, V. Shende, *Sectorial descent for wrapped Fukaya categories*, J. Amer. Math. Soc., 37(2):499–635, 2024.

[4] A. Lauda, A. Licata, D. Manion, *From hypertoric geometry to bordered Floer homology via the $m = 1$ amplituhedron*, Selecta Mathematica, 30(43):1–59, 2024.

[5] Y. Lekili, E. Segal, *Equivariant Fukaya categories at singular values*, arXiv:2304.10969, 2023.

[6] S. Lee, Y. Li, S. Liu, C. Y. Mak, *Fukaya categories of hyperplane arrangements*, Geom. Topo., 29(9), 4841-4909 2025.

Stability in Cubic Metric-Affine Gravity

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Introduction

The quest for a quantum theory of gravity beyond General Relativity (GR) remains one of the most long-standing challenges in theoretical physics. Even though an effective field theory treatment of GR provides a consistent way to compute probability amplitudes at low energies [Str18], its perturbative quantization leads to ultraviolet divergences whose cancellation requires the appearance of infinite counter terms in the gravitational action [tH73, tHV74, DvN74, DTvN74, GS86]. By contrast, a generic extension of the Einstein-Hilbert action of GR with quadratic curvature invariants renders gravity renormalizable [Ste77], but includes second order time derivatives of the metric tensor, which gives rise to ghost instabilities [Woo15]. The latter have in fact been systematically found in a large class of well-motivated theories [DDM93, DGP00, DFS09, MK10, CDL11, Qui19], representing a serious obstacle to the formulation of a theory of quantum gravity beyond GR.

Hence, one of the most relevant aspects to guarantee the viability of any extended theory of gravity concerns stability. This feature is especially relevant in theories that generally include a rich particle spectrum, as is the case of Metric-Affine Gravity (MAG), where the geometrical degrees of freedom correspond not only to the metric tensor but also to the torsion and nonmetricity tensors [BH13, ABGVG24]. Indeed, in line with the theories of gravity described above, the classical formulation of MAG is not free of these issues [YN99, YN02, BJMT20, JCMT22, DJCMT23]. Likewise, the presence of higher-spin fields provided by the torsion and nonmetricity tensors may also lead to further inconsistencies in the quantum regime [Loe08].

Thus, in this article we present a viable solution for these issues in the framework of MAG, showing that the introduction in the gravitational action of cubic order invariants defined from the curvature, torsion and nonmetricity tensors eliminates the well-known ghost instabilities that generally plague the classical formulation of the theory and avoids further inconsistencies that spoil the interaction of higher-spin fields in the quantum regime.

Ghost instabilities in field theories

The most prominent and critical ghost instabilities arising in field theories can be classified as Ostrogradsky, tachyon and gradient instabilities.

In the first case, the core of the issue lies in the nondegeneracy of the Hamiltonian structure of the theory, which can lead to the appearance of a physical mode with a negative energy that is unbounded from below, in virtue of the Ostrogradsky theorem [Ost50]:

Theorem 1. *Let a Lagrangian involve n-th-order finite time derivatives of variables. If $n \geq 2$ and the Lagrangian is nondegenerate with respect to the highest order derivatives, the Hamiltonian of the system depends linearly on a canonical momentum.*

Example 1. An extension of the Klein-Gordon field theory given by the nondegenerate Lagrangian density:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2}\lambda \left(\Box \phi\right)^2 - m^2 \phi^2 \,, \tag{2.1}$$

is characterized by the following Hamiltonian density:

$$\mathcal{H} = q_2 p_1 + \frac{p_2^2}{2\lambda} + \frac{1}{2}\left[(\vec{\nabla} q_1)^2 - \lambda(\vec{\nabla}^2 q_1)^2\right] + m^2 q_1^2 \,, \tag{2.2}$$

which depends linearly on a canonical momentum, thus exhibiting an Ostrogradsky instability.

On the other hand, tachyon and gradient instabilities are signaled in the Lagrangian by the presence of mass terms and spatial derivatives with signs that lead to IR and UV modes with exponential growth, thus violating the energy bounds and breaking down the stability of the theory.

Example 2. A positive sign in the mass term of the Klein-Gordon Lagrangian provides the following solution of the field equations in the Minkowski space-time:

$$\phi\left(t, \vec{x}\right) = \int \left[a_k e^{i(\vec{k}\cdot\vec{x} - \omega_k t)} + a_k^* e^{-\, i(\vec{k}\cdot\vec{x} - \omega_k t)}\right] d^3 k \,, \tag{2.3}$$

with $\omega_k = \sqrt{\vec{k}^2 - m^2}$, which leads to an exponential growth for $\vec{k}^2 < m^2$.

Hence, the presence of ghost instabilities constitutes a fundamental obstacle to the physical viability of any field theory, including the respective extensions of GR.

No-go theorems in the quantum regime

No-go theorems establish rigorous constraints on the types of particles, symmetries and interactions that can consistently coexist in the quantum regime of a field theory. A paradigmatic instance is provided by the Weinberg–Witten theorems, which encapsulate in a particularly transparent way the tension between Lorentz covariance, conserved quantities and the existence of massless higher-spin particles [WW80]:

Theorem 2. *A quantum field theory with a Lorentz covariant current cannot contain massless particles of spin $s > 1/2$ with nonvanishing values of the conserved charge.*

Theorem 3. *A quantum field theory with a Lorentz covariant energy-momentum tensor cannot contain massless particles of spin $s > 1$ with nonvanishing values of the conserved energy-momentum four-vector.*

The key point of the theorems is that the existence of conserved quantities and Lorentz covariance forces correlation functions to yield contradictory results for higher-spin particles:

$$\lim_{p' \to p} \langle p', \pm s | J^{\mu}(t, 0) | p, \pm s \rangle = \frac{q p^{\mu}}{p^0 (2\pi)^3} \neq 0 \,, \tag{3.1}$$

$$\lim_{p' \to p} \langle p', \pm s | J^{\mu}(t, 0) | p, \pm s \rangle = 0 \,, \text{ for } s > 1/2 \,, \tag{3.2}$$

and

$$\lim_{p' \to p} \langle p', \pm s | T^{\mu\nu}(t, 0) | p, \pm s \rangle = \frac{q p^{\mu} p^{\nu}}{p^0 (2\pi)^3} \neq 0 \,, \tag{3.3}$$

$$\lim_{p' \to p} \langle p', \pm s | T^{\mu\nu}(t, 0) | p, \pm s \rangle = 0 \,, \text{ for } s > 1 \,. \tag{3.4}$$

Accordingly, this quantum inconsistency implies that, under the aforementioned assumptions, higher-spin particles must be massive. Alternatively, the conclusions of the theorems may be evaded by relaxing any of their assumptions.

Stability in quadratic MAG

The identification of the ghost instabilities arising in quadratic MAG can be readily established by analyzing the minimal extension of the Einstein-Hilbert action provided by the torsion and nonmetricity fields, which includes quadratic invariants of order $\tilde{R}^2$, T^2 and Q^2, where $\tilde{R}$, T and Q refer to the curvature, torsion and nonmetricity tensors:

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = 2\partial_{[\mu|}\tilde{\Gamma}^{\lambda}{}_{\rho|\nu]} + 2\tilde{\Gamma}^{\lambda}{}_{\sigma[\mu|}\tilde{\Gamma}^{\sigma}{}_{\rho|\nu]} \,, \quad T^{\lambda}{}_{\mu\nu} = 2\tilde{\Gamma}^{\lambda}{}_{[\mu\nu]} \,, \quad Q_{\lambda\mu\nu} = \tilde{\nabla}_{\lambda} g_{\mu\nu} \,. \tag{4.1}$$

By performing a post-Riemannian expansion and an irreducible decomposition of these tensors under the four-dimensional pseudo-orthogonal group [McC92, BGV23, BGVS24], it can be shown that the quadratic action then generally contains pathological terms of the form

$$(\nabla X)^2 \,, \nabla X \nabla Y, X^2 \nabla Y, XY\nabla X, XY\nabla Z, RX^2, RXY, R\nabla X, \text{ with } X \neq Y, \tag{4.2}$$

which signal the presence of Ostrogradsky instabilities.

Likewise, the kinetic part of the vector and axial sectors of the action takes the general form

$$\mathcal{L}_{FF} = -\frac{1}{4}\kappa_{XY} F^{(X)}_{\mu\nu} F^{(Y)\mu\nu} \,, \quad F^{(X)} = (F^{(S)}, F^{(T)}, F^{(W)}, F^{(\Lambda)}) \,, \tag{4.3}$$

with $F^{(X)}_{\mu\nu} = 2\partial_{[\mu}X_{\nu]}$. However, it turns out that the kinetic matrix κ_{XY} fails to have strictly positive eigenvalues, which also involves the presence of physical modes with negative kinetic energies.

Cubic MAG and the restoration of stability

As a first step in addressing the stability issue of MAG, it is important to take into account that the mathematical structure of the theory does not single out a unique Lagrangian, but the dynamics of the metric, torsion and nonmetricity tensors can be implemented by including higher order invariants in the action.

In this regard, cubic order scalars constructed from the curvature, torsion and nonmetricity tensors are especially relevant, since they respect the gauge invariance of the theory and contain interaction terms, such as those given by Expression (4.2), in their post-Riemannian expansions. Hence, they can potentially cancel the ghost instabilities of the quadratic action by fine-tuning of the Lagrangian coefficients.

In general, there are three different branches, which can be used for this task.
Cubic invariants from curvature and torsion: The first branch consists of 46 even-parity gauge invariants of order $\tilde{R}T^2$, constructed from the curvature and torsion tensors.
Cubic invariants from curvature and nonmetricity: The second branch is given by 61 even-parity gauge invariants of order $\tilde{R}Q^2$, constructed from the curvature and nonmetricity tensors.
Cubic invariants from curvature, torsion and nonmetricity: The third branch mixes the curvature, torsion and nonmetricity tensors, providing 102 even-parity gauge invariants of order $\tilde{R}TQ$.

Overall, the respective cubic interactions provided by the gauge invariants above can indeed cancel all of the instabilities of the form:

$$X^2\nabla Y, XY\nabla X, XY\nabla Z, RX^2, RXY, \text{ with } X \neq Y, \tag{5.1}$$

which are present in the quadratic theory. The resulting action is then less constrained and a further restriction of the Lagrangian coefficients of the quadratic part enables the cancellation of the remaining pathological terms:

$$(\nabla X)^2, \nabla X\nabla Y, R\nabla X, \text{ with } X \neq Y, \tag{5.2}$$

turning the mentioned ghost modes of quadratic MAG into healthy fields.
Furthermore, in contrast to quadratic MAG [CGV17, CGV18, BCGV23], the field equations of the cubic action provide Reissner-Nordström-like black holes with massive tensor modes, thus avoiding the no-go theorems that prevent a consistent interaction of the higher-spin modes of torsion and nonmetricity in the quantum regime.

References

[ABGVG24] K. Aoki, S. Bahamonde, J. Gigante Valcarcel, and M. A. Gorji. Cosmological perturbation theory in metric-affine gravity. *Phys. Rev. D*, 110(2):024017, 2024.

[BCGV23] S. Bahamonde, J. Chevrier, and J. Gigante Valcarcel. New black hole solutions with a dynamical traceless nonmetricity tensor in Metric-Affine Gravity. *JCAP*, 02:018, 2023.

[BGV23] S. Bahamonde and J. Gigante Valcarcel. Algebraic classification of the gravitational field in Weyl-Cartan spacetimes. *Phys. Rev. D*, 108(4):044037, 2023.

[BGVS24] S. Bahamonde, J. Gigante Valcarcel, and J. M. M. Senovilla. Algebraic classification of the gravitational field in general metric-affine geometries. *Phys. Rev. D*, 110(12):124053, 2024.

[BH13] M. Blagojević and F. W. Hehl. *Gauge Theories of Gravitation: A Reader with Commentaries*. World Scientific, Singapore, 2013.

[BJMT20] J. Beltrán Jiménez and F. J. Maldonado Torralba. Revisiting the stability of quadratic Poincaré gauge gravity. *Eur. Phys. J. C*, 80(7):611, 2020.

[CDL11] S. Capozziello and M. De Laurentis. Extended Theories of Gravity. *Phys. Rept.*, 509:167–321, 2011.

[CGV17] J. A. R. Cembranos and J. Gigante Valcarcel. New torsion black hole solutions in Poincaré gauge theory. *JCAP*, 01:014, 2017.

[CGV18] J. A. R. Cembranos and J. Gigante Valcarcel. Extended Reissner–Nordström solutions sourced by dynamical torsion. *Phys. Lett. B*, 779:143–150, 2018.

[DDM93] T. Damour, S. Deser, and J. G. McCarthy. Nonsymmetric gravity theories: Inconsistencies and a cure. *Phys. Rev. D*, 47:1541–1556, 1993.

[DFS09] A. De Felice and T. Suyama. Vacuum structure for scalar cosmological perturbations in Modified Gravity Models. *JCAP*, 06:034, 2009.

[DGP00] G. R. Dvali, G. Gabadadze, and M. Porrati. A Comment on brane bending and ghosts in theories with infinite extra dimensions. *Phys. Lett. B*, 484:129–132, 2000.

[DJCMT23] A. Delhom, A. Jiménez-Cano, and F. J. Maldonado Torralba. Instabilities in Field Theory: A Primer with Applications in Modified Gravity. SpringerBriefs in Physics. Springer, 2023.

[DTvN74] S. Deser, H.-S. Tsao, and P. van Nieuwenhuizen. Nonrenormalizability of Einstein-Yang-Mills interactions at the one-loop level. *Phys. Lett. B*, 50:491–493, 1974.

[DvN74] S. Deser and P. van Nieuwenhuizen. One-loop divergences of quantized Einstein-Maxwell fields. *Phys. Rev. D*, 10:401, 1974.

[GS86] M. H. Goroff and A. Sagnotti. The ultraviolet behavior of Einstein gravity. *Nucl. Phys. B*, 266:709–736, 1986.

[JCMT22] A. Jiménez-Cano and F. J. Maldonado Torralba. Vector stability in quadratic metric-affine theories. *JCAP*, 09:044, 2022.

[Loe08] Florian Loebbert. The Weinberg-Witten theorem on massless particles: An Essay. *Annalen Phys.*, 17:803–829, 2008.

[McC92] J. D. McCrea. Irreducible decompositions of non-metricity, torsion, curvature and Bianchi identities in metric-affine spacetimes. *Class. Quant. Grav.*, 9:553–568, 1992.

[MK10] R. Maartens and K. Koyama. Brane-World Gravity. *Living Rev. Rel.*, 13:5, 2010.

[Ost50] M. Ostrogradsky. Mémoires sur les équations différentielles, relatives au problème des isopérimètres. *Mem. Acad. St. Petersbourg*, 6(4):385–517, 1850.

[Qui19] I. Quiros. Selected topics in scalar–tensor theories and beyond. *Int. J. Mod. Phys. D*, 28(07):1930012, 2019.

[Ste77] K. S. Stelle. Renormalization of Higher Derivative Quantum Gravity. *Phys. Rev. D*, 16:953–969, 1977.

[Str18] A. Strominger. *Lectures on the infrared structure of gravity and gauge theory*. Princeton University Press, 2018.

[tH73] G. 't Hooft. An algorithm for the poles at dimension four in the dimensional regularization procedure. *Nucl. Phys. B*, 62:444–460, 1973.

[tHV74] G. 't Hooft and M. J. G. Veltman. One-loop divergencies in the theory of gravitation. *Ann. Inst. H. Poincare Phys. Theor. A*, 20:69–94, 1974.

[Woo15] R. P. Woodard. Ostrogradsky's theorem on Hamiltonian instability. *Scholarpedia*, 10(8):32243, 2015.

[WW80] Steven Weinberg and Edward Witten. Limits on Massless Particles. *Phys. Lett. B*, 96:59–62, 1980.

[YN99] H.-J. Yo and J. M. Nester. Hamiltonian analysis of Poincaré gauge theory scalar modes. *Int. J. Mod. Phys. D*, 8:459–479, 1999.

[YN02] H.-J. Yo and J. M. Nester. Hamiltonian analysis of Poincaré gauge theory: higher spin modes. *Int. J. Mod. Phys. D*, 11:747–780, 2002.

Lagrangian Subvarieties from Noncommutative Curves to Surfaces: Gushel-Mukai Varieties and Their Moduli Spaces

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Introduction and historical context

The study of Lagrangian subvarieties represents one of the most fruitful intersections of symplectic geometry, algebraic geometry, and mathematical physics. A Lagrangian subvariety L in a symplectic manifold (M, ω) satisfies the fundamental conditions $\omega|_L = 0$ (isotropic condition) and $\dim L = \frac{1}{2} \dim M$ (maximal dimension). In algebraic geometry, particularly in the context of hyperkähler manifolds, Lagrangian subvarieties play a crucial role in understanding the geometry, topology, and moduli theory of these spaces. The classical prototype arises from K3 surfaces. Let S be an algebraic K3 surface equipped with a holomorphic symplectic form $\omega_S \in \mathrm{H}^{2,0}(S)$. A smooth projective curve $C \subset S$ is a **Lagrangian subvariety** since $\omega_S|_C = 0$ and $\dim C = 1 = \frac{1}{2} \dim S$. The Hilbert scheme of two points $S^{[2]}$ is a hyperkähler manifold, and when $C \subset S$ is Lagrangian, the symmetric square $\mathrm{Sym}^2(C)$ embeds as a Lagrangian subvariety in $S^{[2]}$, as well. Recent developments in derived categories and noncommutative geometry have revealed deep generalizations of this classical picture. The pioneering work of Kuznetsov on semiorthogonal decompositions, particularly for Fano varieties, has led to the revolutionary concept of "noncommutative algebraic geometry," where categories play the role of spaces. This paper provides a comprehensive exploration of how the classical Lagrangian construction generalizes to this noncommutative setting, with particular emphasis on Gushel-Mukai varieties and their Bridgeland moduli spaces in the Kuznetsov components.

Lagrangian subvarieties and classical examples

Definition 1. Let (M, ω) be a holomorphic symplectic manifold of dimension $2n$. A subvariety $L \subset M$ is **Lagrangian** if:

(1) $\omega|_L = 0$ (isotropic condition)

(2) $\dim L = n$ (maximal dimension)

A **K3 surface** is a compact complex surface S satisfying:

(1) The canonical bundle is trivial: $K_S \cong \mathcal{O}_S$

(2) The irregularity vanishes: $\mathrm{H}^1(S, \mathcal{O}_S) = 0$

Every K3 surface admits a unique (up to scalar) non-degenerate holomorphic 2-form $\omega_S \in \mathrm{H}^0(S, \omega_S)$. This form makes S into a holomorphic symplectic manifold. Let $C \subset S$ be a smooth curve on a K3 surface.The The curve C is Lagrangian if and only if $\omega_S|_C = 0$, which automatically holds for any curve on a K3 surface since the restriction of the global holomorphic 2-form to any curve becomes 0.

Hilbert scheme on K3 surface and its Lagrangian subvariety

The Hilbert scheme of points $S^{[n]}$ parameterizes length-n subschemes of S. For a K3 surface S, $S^{[n]}$ inherits rich geometric structures:

Theorem 2 ([Bea83])**.** *The Hilbert scheme $S^{[n]}$ of a K3 surface S is an irreducible holomorphic symplectic manifold. The symplectic form on $S^{[n]}$ is induced from the symplectic form on S via the natural correspondence.*

Given a curve $C \subset S$, we can consider the symmetric n-th power $C^{[n]} \cong C^{(n)} \subset S^{[n]}$ parameterizing subschemes supported on C. Then $C^{(n)}$ is a Lagrangian subvariety of $S^{[n]}$.

Noncommutative geometry and Kuznetsov components

Semiorthogonal decompositions: foundations

The transition from classical to noncommutative geometry is facilitated by the sophisticated theory of semiorthogonal decompositions [Kuz14], which provides a categorical analogue of the decomposition of spaces into simpler components.

Definition 3. Let $\mathcal{T}$ be a triangulated category. A **semiorthogonal decomposition**

$$\mathcal{T} = \langle \mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n \rangle$$

is a sequence of full triangulated subcategories $\mathcal{A}_i$ such that:

(1) $\mathrm{Hom}(\mathcal{A}_j, \mathcal{A}_i) = 0$ for $i < j$

(2) The smallest triangulated subcategory containing all $\mathcal{A}_i$ is $\mathcal{T}$

(3) Each inclusion $\mathcal{A}_i \hookrightarrow \mathcal{T}$ admits left and right adjoints

Kuznetsov's fundamental insight was that for many Fano varieties, the derived category $D^b(X)$ admits a semiorthogonal decomposition where one component has special properties that mimic the derived category of a curve or surface.

Gushel-Mukai varieties: definition and properties

Definition 4. A **Gushel-Mukai (GM)** n**-fold** is a smooth n-dimensional intersection:

$$X = \mathrm{Gr}(2,5) \cap Q \cap \mathbb{P}^{n+4},$$

where Q is a quadric hypersurface. A GM fourfold has dimension 4, and its hyperplane section is a GM threefold.

GM fourfolds are particularly interesting because their derived categories exhibit remarkable structures that generalize the classical theory of K3 surfaces:

Theorem 5 ([KP18]). *For a GM fourfold X, there is a semiorthogonal decomposition:*

$$D^b(X) = \langle \mathcal{K}u(X), \mathcal{O}_X, \mathcal{U}_X^\vee, \mathcal{O}_X(H), \mathcal{U}_X^\vee(H) \rangle.$$

The component $\mathcal{K}u(Y)$ is a K3 category i.e., its Serre functor satisfies $S_{\mathcal{K}u(X)} = [2]$ and the Hochschild cohomology $\mathrm{HH}^\bullet(\mathcal{K}u(X))$ is the same as a K3 surface.

For a GM threefold Y, as the hyperplane section of X, we have the analogous decomposition:

$$D^b(Y) = \langle \mathcal{K}u(Y), \mathcal{O}_Y, \mathcal{U}_Y^\vee \rangle,$$

where $\mathcal{K}u(Y)$ is an Enriques category. Since the numerical Grothendieck group $\mathcal{N}(\mathcal{K}u(Y)) \cong \mathbb{Z}^2$ is of rank two, and the Hochschild homology $\mathrm{HH}_1(\mathcal{K}u(Y)) \cong \mathrm{H}^{2,1}(Y) \cong k^{10}$, the Kuznetsov component $\mathcal{K}u(Y)$ could be regarded as a **noncommutative curve** of genus 10, while $\mathcal{K}u(X)$ is a **noncommutative K3 surface**. The hyperplane inclusion $i : Y \hookrightarrow X$ induces an adjoint pair of functors [FGLZ25, Lemma 4.4]

$$i^* : \mathcal{K}u(X) \hookrightarrow \mathcal{K}u(Y), \mathrm{pr}_X \circ i_* : \mathcal{K}u(Y) \to \mathcal{K}u(X),$$

where $\mathrm{pr}_X : D^b(X) \to \mathcal{K}u(X)$ is the projection functor with respect to the semi-orthogonal decomposition above. This is the **noncommutative** analogue of a curve embedded in a K3 surface.

Bridgeland moduli spaces: theory and constructions

Stability conditions

Bridgeland stability conditions [Bri07] provide a powerful framework for constructing moduli spaces of objects in triangulated categories, generalizing the classical notion of stability for sheaves.

Definition 6. A **Bridgeland stability condition** $\sigma = (Z, \mathcal{A})$ on a triangulated category $\mathcal{D}$ consists of:

- A heart $\mathcal{A}$ of a bounded t-structure on $\mathcal{D}$
- A stability function $Z : K_0(\mathcal{A}) \to \mathbb{C}$ such that for all $0 \neq E \in \mathcal{A}$, $Z(E) \in \mathbb{H} = \{z \in \mathbb{C} : z = re^{i\pi\phi}, r > 0, 0 < \phi \leq 1\}$
- The Harder-Narasimhan property holds for σ

For a primitive Mukai vector $v \in K_{\text{num}}(\mathcal{D})$, we can consider the moduli space $\mathcal{M}_\sigma(v)$ of σ-stable objects with Mukai vector v.

Moduli spaces for Kuznetsov components

For GM varieties, the Bridgeland moduli spaces associated to Kuznetsov components exhibit particularly nice properties that generalize the classical theory of moduli spaces on K3 surfaces:

Theorem 7 ([PPZ19]). *Let X be a GM fourfold and $v \in K_{num}(\mathcal{K}u(X))$ a primitive Mukai vector. For a generic stability condition σ on $\mathcal{K}u(X)$, the moduli space $\mathcal{M}_\sigma(\mathcal{K}u(X), v)$ is a smooth, projective hyperkähler manifold of dimension:*

$$\dim \mathcal{M}_\sigma(\mathcal{K}u(X), v) = \langle v, v \rangle + 2.$$

Similarly, for a GM threefold Y and primitive Mukai vector $w \in K_{\text{num}}(\mathcal{K}u(Y))$, the moduli space $\mathcal{M}_{\sigma'}(\mathcal{K}u(Y), w)$ is a smooth projective variety whenever it is non-empty for general $Y \subset X$. Motivated by the classical Lagrangian embedding $C^{(n)} \hookrightarrow S^{[n]}$ of a hyperkahler variety, we construct a Lagrangian subvariety of $\mathcal{M}_\sigma(\mathcal{K}u(X), v)$ via the Bridgeland moduli space over **noncommutative curve** $\mathcal{K}u(Y)$, where $Y \hookrightarrow X$ is the hyperplane section.

The Lagrangian construction: main theorem and examples

Statement of main theorem

We now state the main result in its full generality:

Theorem 8. *[FGLZ25, Theorem 1.1] Let X be a general GM fourfold, and $j\colon Y \hookrightarrow X$ be a smooth hyperplane section. Given $\sigma_X \in \mathrm{Stab}^\circ(\mathcal{K}u(X))$ and a Serre-invariant stability condition σ_Y on $\mathcal{K}u(Y)$. If $E \in \mathcal{K}u(Y)$ is a σ_Y-semistable object, then $\mathrm{pr}_X(j_*E)$ is σ_X-semistable.*

The above theorem can be viewed as a noncommutative analogue of the stability of push-forward/pull-back objects for the embedding of a curve into a K3 surface. Using

Theorem above we can construct families of Lagrangian subvarieties as follows. We denote by $M^X_{\sigma_X}(a,b)$ (resp. $M^Y_{\sigma_Y}(a,b)$) the moduli space that parameterizes S-equivalence classes of σ_X-semistable (resp. σ_Y-semistable) objects of class $a\Lambda_1 + b\Lambda_2$ (resp. $a\lambda_1 + b\lambda_2$) in $\mathcal{K}u(X)$ (resp. $\mathcal{K}u(Y)$). In this case, there is a morphism

$$M^Y_{\sigma_Y}(a,b) \to M^X_{\sigma_X}(a,b), \quad E \mapsto \mathrm{pr}_X(j_*E)$$

which is finite and unramified when $Y \subset X$ is a general hyperplane section, and we show that its image is a Lagrangian subvariety of the hyperkähler manifold $M^X_{\sigma_X}(a,b)$. Therefore, as we vary the hyperplane section $Y \subset X$, we obtain a family of Lagrangian subvarieties of $M^X_{\sigma_X}(a,b)$, which can be arranged into a Lagrangian family as detailed below.

Theorem 9. *[FGLZ25, Theorem 1.3] Let a,b be a pair of coprime integers and X be a general GM fourfold. Then for any $\sigma_X \in \mathrm{Stab}^\circ(\mathcal{K}u(X))$ generic with respect to $a\Lambda_1 + b\Lambda_2$, there is a Lagrangian family of $M^X_{\sigma_X}(a,b)$ over an open dense subset of $|\mathcal{O}_X(H)|$.*

Examples and explicit descriptions

For a,b to be small integer numbers, the Lagrangian embedding

$$M^Y_{\sigma_Y}(a,b) \to M^X_{\sigma_X}(a,b)$$

recovers the classical construction.

(1) $(a,b) = (0,1)$ or $(1,0)$, the moduli space $M^X_{\sigma_X}(a,b)$ is a double EPW sextic or a double dual EPW sextic, which are four-dimensional hyperkahler varieties, and $M^Y_{\sigma}(a,b)$ are double EPW surface or double dual EPW surface, which are Lagrangian subvarieties, which were studied by Iliev-Manivel.

(2) $(a,b)=(1,\pm1)$, the moduli space $M^X_{\sigma_X}(a,b)$ is a double EPW cube and it is a MRC quotient of the twisted cubics on the Gushel-Mukai fourfold X, while $M^Y_{\sigma_Y}(a,b)$ is a three dimensional Lagrangian subvariety which is the divisorial contraction of the twisted cubics on the Gushel-Mukai threefold Y.

An analogue of Theorem 8 has been proved in [LLPZ24, Theorem 1.8] for cubic fourfold X and its hyperplane section Y as a cubic threefold. Then

(1) $(a,b)=(0,1),(1,0),(1,\pm1),(2,\pm1),(1,\pm2)$, the moduli space $M^X_{\sigma_X}(a,b)$ is the Fano variety of lines on cubic fourfold X, which is a four-dimensional hyperkahler variety and $M^Y_{\sigma_Y}(a,b)$ is the Fano surface of lines on Y, as a Lagrangian subvariety, which is studied intensively by Voisin.

(2) $(a,b)=(2,\pm1),(1,\pm2)$, the moduli space $M^X_{\sigma_X}(a,b)$ is the LLsVS eightfold and $M^Y_{\sigma_Y}(a,b)$ is the four-dimensional moduli space coming from the twisted cubics on cubic threefold Y, as a Lagrangian subvariety.

References

[Bea83] Arnaud Beauville. Some remarks on Kähler manifolds with $c_1=0$. In *Classification of algebraic and analytic manifolds (Katata, 1982)*, volume 39 of *Progr. Math.*, pages 1–26. Birkhäuser Boston, Boston, MA, 1983.

[Bri07] T. Bridgeland. Stability conditions on triangulated categories. *Ann. of Math. (2)*, 166(2):317–345, 2007.

[FGLZ25] Soheyla Feyzbakhsh, Hanfei Guo, Zhiyu Liu, and Shizhuo Zhang. Lagrangian families of bridgeland moduli spaces from gushel-mukai fourfolds. *Compositio Mathematica*, 2025. To appear.

[KP18] Alexander Kuznetsov and Alexander Perry. Derived categories of gushel–mukai varieties. *Compositio Mathematica*, 154(7):1362–1406, 2018.

[Kuz14] Alexander Kuznetsov. Semiorthogonal decompositions in algebraic geometry. *arXiv preprint*, 2014.

[LLPZ24] Chunyi Li, Yinbang Lin, Laura Pertusi, and Xiaolei Zhao. Higher dimensional moduli spaces on kuznetsov components of fano threefolds. *arXiv preprint*, 2024.

[PPZ19] Alexander Perry, Laura Pertusi, and Xiaolei Zhao. Stability conditions and moduli spaces for Kuznetsov components of Gushel–Mukai varieties. To appear in Geometry and Topology, arXiv:1912.06935, 2019.

Interview

Jihyeon Lee

Florian Michael Zeiser

Sungkyung Kang

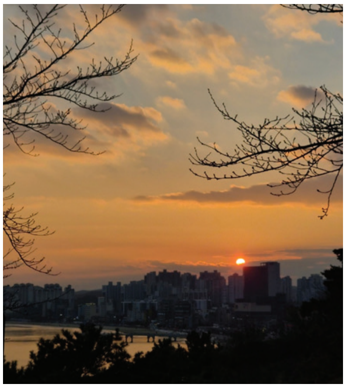
Yat-Hin Suen

Jihyeon Lee

Research member since March 2025

How's your life in CGP / Pohang / Korea?

Life in CGP and Pohang has been both comfortable and inspiring for me. I naturally enjoy a quiet and steady environment, and Pohang offers exactly that kind of space. The calm atmosphere allows me to focus deeply on my research without unnecessary distraction. At the same time, being part of CGP has been especially meaningful. The institute provides a supportive and well-structured environment where I can fully concentrate on developing my work. The balance between intellectual focus and the gentle rhythm of daily life in Pohang helps me remain productive while also giving me the space to rest and recharge.



What is the attractive points of mathematics/mathematical physics? What made you decide to be a mathematician/mathematical physicist?

I still remember the shock I felt when I first learned that something squared could become negative. It made me realize that what I had known might not be the whole story. From that moment, a curiosity about the larger world of mathematics began to grow within me. I became fascinated by the questions, "What else do I not know yet?" and "What more will I be able to understand in the future?". These thoughts filled me with curiosity and anticipation, and they gradually drew me deeper and deeper into the world of mathematics. In university, I was especially drawn to differential geometry because of its visual nature. I was fascinated by how abstract ideas could almost be seen, as if I were learning to perceive an invisible world. Together with my ongoing curiosity ultimately led me to pursue the path of becoming a mathematician.

What is your current research-related interest? Please tell us about your research.

My current research interest lies in Lagrangian mean curvature flow, a geometric evolution equation for Lagrangian submanifolds in Kahler manifolds. I am particularly interested in understanding how such submanifolds evolve toward canonical geometric structures, such as special Lagran gians, and analyzing the formation of singularities and stability phenomena along the flow. Through this study, I aim to clarify the analytic and geometric mechanisms governing higher-codimension geometric evolution. This topic naturally connects symplectic geometry with geometric analysis, and contributes to a deeper understanding of the interplay between curvature, topology and symplectic structures.

Another direction of my research concerns rigidity phenomena near the event horizon of black holes from a geometric perspective. In general relativity, spacetime is modeled as a Lorentzian manifold satisfying the Einstien equations, and a black hole is defined as a region of such a spacetime characterized by the presence of an event horizon. I am particularly interested in understanding when the local geometry near an event horizon is uniquely determined by physical quantities such as mass, charge, and the cosmological constant. By analyzing geometric invariants and quasi-local mass functionals, I investigate conditions under which a spacetime must coincide with standard model solutions, such as Schwarzschild or Reissner-Nordstrum metrics. This work lies at the intersection of differential geometry and mathematical relativity, aiming to clarify how curvature and physical constraints enforce rigidity in extreme gravitational regimes.

What are you interested in recently? Please share something about you, not math. (hobby, art, episode, travel story and etc.)

Recently, I have really gotten into cooking. I enjoy preparing delicious meals at home, plating them nicely, and taking the time to truly savor the food. The whole process - from chopping ingredients to sitting down for a warm meal - helps me slow down and reset. I like experimenting with different dishes, from pasta and shabu-shabu to suyuk, seaweed soup, and traditional Korean food like doenjang-jjigae. Cooking has become a small but meaningful ritual for me - a way to relax, stay healthy, and recharge outside of research.

We'd like to hear about your dream and future plans. Do you have a role model or a philosophy of life?

If I were to describe my philosophy in life in one sentence, it would be this: live sincerely and consistently. I believe that meaningful achievements come not from brief moments of inspiration, but from steady effort sustained over time. Even when progress feels slow, taking one faithful step after another ultimately builds something solid and lasting. Looking ahead, I hope to continue walking my path with clear purpose and quiet persistence. Rather than rushing, I want to grow patiently, taking responsibility for the work I choose and moving forward with steady passion.

Florian Michael Zeiser

Research member since October 2025

How's your life in CGP / Pohang / Korea?

I enjoy the institute's atmosphere, the variety of seminars is great, and the Wednesday lunch talks are a nice way to glimpse others' interests and make connections. I also like the food. Although different from what I'm used to, exploring the dishes has been a pleasure. It is my first time living so close to the sea, which is wonderful. As a newcomer, it is always a challenge to find a community outside work, and the language barrier doesn't make it easier. I've started taking Korean classes, which I hope will help in that regard. Overall, it has been a very enjoyable adventure so far.

What is the attractive points of mathematics/mathematical physics? What made you decide to be a mathematician/mathematical physicist?

I was a stepwise process really. I was always good at math in school, but initially I was more drawn to chemistry. The idea of mixing substances and seeing dramatic reactions appealed to me as a kid. My father steered me toward a more technical high school, which ended up giving me a lot of math and applied physics. When I started university, I wasn't sure whether to choose math or physics, so I began in mathematics while taking physic lectures as electives. Over time I fell more and more in love with the precision of mathematics. Nowadays, I see mathematics as the language of natural science. For a mathematician a differential equation for example, doesn't carry a fixed meaning, but in across other areas such as biology, engineering, or physics this equation may describe different processes by assigning a meaning to the variables. It allows us to have a basic understanding of these process in very different areas, which I find really fascinating. It's a universal language can open conversations with many different fields.

What is your current research-related interest? Please tell us about your research.

I study Poisson structures, focusing on normal forms, deformation problems, and the cohomology that describes these deformations infinitesimally. I like how Poisson geometry connects different areas such as foliation theory, symplectic geometry, and Lie theory, and how finding questions in this field naturally link methods from those subjects. Being at the institute and talking with colleagues has also inspired me to think about potential applications of holomorphic curve theory in Poisson settings and physical systems where Poisson theory might be relevant.

What are you interested in recently? Please share something about you, not math. (hobby, art, episode, travel story and etc.)

I really enjoy salsa dancing. Since I haven't found a Latin community in Pohang, I often go to Busan on weekends for dance socials. Besides dancing I like sports in general and work out regularly, so it's convenient to have a gym right in the building.

I've also been exploring Korea: I visited for example Jeonju to see the hanok village and try Jeonju bibimbap; Seoul, where I saw the digital media art exhibition at the National Museum, toured Gyeongbokgung Palace, and learned about the wartime period at the Seodaemun Prison Museum; and Gyeongju, to see its historic sites. As part of the Korean language class at PICL, we visited Jukdo Market to prepare our own Charye for Seollal. I've really enjoyed learning more about Korean history and culture.

We'd like to hear about your dream and future plans. Do you have a role model or a philosophy of life?

Above all, I hope to find more (geographical) stability soon. I've been fortunate to hold postdoctoral positions on three continents, which has been an invaluable experience, but it also requires constantly rebuilding your personal life. I'm aiming to secure a more permanent position, ideally a professorship in mathematics, when the right opportunity arises.

I don't have a single role model, I think everyone offers traits worth learning from. That said, several people have shaped me profoundly. First and foremost, my family and some close friends. Academically, I've been fortunate to have excellent mentors throughout my life really. Most notably Will Merry, my master's thesis supervisor, who inspired me to continue in mathematics, and Ioan Marcut, my PhD supervisor, who created an open and supportive environment throughout the highs and lows of my PhD.

Sungkyung Kang

Assistant Professor at University of Cambridge
(Research member from August 2020 to August 2023)

How's your life and work after CGP?

I worked at CGP from August 2020 to August 2023 and am currently working as a tenure-track assistant professor and a Royal Society University Research Fellow at Department of Pure Mathematics and Mathematical Statistics, University of Cambridge. I am also an official fellow at St Catharine's College, University of Cambridge.

There are so many prominent mathematicians in Cambridge. Our head of department, Professor Ivan Smith, is a prominent figure in symplectic geometry. Professor Oscar Randal-Williams, whose office is right next to mine, is also a prominent figure in homotopy theory. Being in the same department as them feels like I am living my dream; they inspire me to try harder and strive to be a better mathematician.

Currently I am trying to adjust to the life of a faculty, as I now have many more duties than when I was a postdoc. I must prepare for lectures, participate in various departmental duties (like interviewing PhDs, postdocs, and attending departmental meetings). While it is my first year here, which means that I still have much less administrative duties compared to more senior professors here, adjusting to this new lifestyle is quite a nontrivial task. But I am trying my best.

What is the attractive points of mathematics/mathematical physics? What made you decide to be a mathematician/mathematical physicist?

I always thought mathematics was beautiful and elegant, but that did not necessarily mean that I wanted to be a serious mathematician. For me, attending the first math lecture (which was an undergraduate analysis course) in my first day as an undergraduate student at KAIST made me to pursue a career in mathematics, as it made me feel for the first time that mathematics is not just an interesting puzzle, but rather a profound language that can rigorously describe my intuitions. Then, when I was graduating from KAIST, I read the book "Instantons and 4-manifolds" for the first time to learn about Donaldson's diagonalization theorem; it was so amazing that I decided to study 4-manifolds.

What is your current research-related interest? Please tell us about your research.

I am interested in various questions involving 4-dimensional manifolds and related objects. For example, given a knot, does it bound a disk in the 4-ball? Given an exotic smooth structure on a 4-manifold, how many stabilizations do we need to make it non-exotic? How about the same question for exotic diffeomorphism of 4-manifolds? To answer these questions, I use various techniques from Floer theory, gauge theory, and homotopy theory. Most recently, in a joint work with JungHwan Park and Masaki Taniguchi, I proved that there exists an exotic diffeomorphism of a 4-manifold (with boundary) which stays exotic after two stabilizations.

One of my lifetime goals is to show that, for any integer n, there exist exotic structures and exotic diffeomorphisms (on 4-manifold, possibly with boundary) that stay exotic after n stabilizations.

What are you interested in recently? Please share something about you, not math. (hobby, art, episode, travel story and etc.)

I am always interested in travelling; apart from (a lot of) research trips, I also enjoy travelling around UK and Europe and looking at various historical buildings and artifacts. Even when I was in CGP, I did trips to various nearby cities, including Busan (my favorite!), Gyeongju, Ulsan, Tongyeong, and Jinju. After moving to Cambridge, I have visited nearby towns, like Ely, Bury St Edmunds, and Norwich, whenever I had free time during weekends. Currently I am planning to do a hiking in Cheddar Gorge right after the end of this semester.

We'd like to hear about your dream and future plans. Do you have a role model or a philosophy of life?

Mathematically, my dream is to prove two things: unbounded instability of 4-manifolds and smooth non-sliceness of the Whitehead double of the figure-eight knot. Non-mathematically, my dream is to be a happy and productive mathematician for the rest of my life.

My philosophy of life is to stay happy. To be happy, I need to be a productive mathematician. Also, to be a productive mathematician, I need to be happy. Happiness and productivity affect each other!

Is there anything you want to tell to younger researchers?

Be happy. Enjoy your life. Try your best and be brave in mathematics. Also, if you happen to be in CGP, enjoy mulhoe as much as you can. You cannot find mulhoe of that quality outside Pohang.

Yat-Hin Suen

Assistant Professor at National Cheng Kung University
(Research member from August 2018 to August 2023)

How's your life and work after CGP?

After completing my position at CGP, I moved to KIAS for another postdoctoral appointment. However, I stayed there for only a year, as I received a tenure-track offer from National Cheng Kung University in Tainan, a city in southern Taiwan. Tainan is well known for its local cuisine and relatively low cost of living compared to Taipei, so I have been quite happy with my life here. That said, Taiwan's subtropical climate sometimes makes me miss the beautiful autumn—and even winter—weather in Pohang. In addition to research, I now teach three courses per year. Although teaching reduces the time I can devote to research and travel, I genuinely enjoy sharing knowledge with the younger generation. It is both a responsibility and a rewarding experience.

What is the attractive points of mathematics/mathematical physics? What made you decide to be a mathematician/mathematical physicist?

Believe it or not, I was very poor at mathematics as a child—I even failed several math exams. My turning point came in middle school, when I first encountered the Euler characteristic of surfaces (at the time, only for spheres). I learned that any closed convex polyhedron in three-dimensional Euclidean space has Euler characteristic 2: no matter how the polyhedron looks, if one counts the number of vertices, subtracts the number of edges, and then adds the number of faces, the result is always 2. I was deeply fascinated by this mysterious and universal phenomenon. How could such different shapes all obey the same simple formula? I was captivated by this question and it gradually drew me toward mathematics, especially geometry. Later, my high school teacher recommended me for a summer program at The Chinese University of Hong Kong (where I would eventually earn my PhD). The program focused on differential geometry and hyperbolic geometry. During that time, I came to realize that geometry and physics are profoundly interconnected. The structures I was learning in geometry were not merely abstract—they resonated with fundamental ideas in physics. Motivated by a desire to understand this deep relationship, I decided to pursue a career in mathematics, more specifically in mathematical physics. Although my training in physics has never been as strong as my background in mathematics, physics has always remained a guiding inspiration for my research. The interplay among geometry and physics reveals deeper structures underlying mathematics, and is what drew me to this field and continues to inspire my work today.

What is your current research-related interest? Please tell us about your research.

My current research focuses on the geometric aspects of mirror symmetry and its applications. Mirror symmetry, originally discovered by string theorists, predicts the existence of pairs of spaces with different geometries but equivalent physical theories.

Mathematically, it builds a bridge between symplectic geometry (the A-side) and algebraic geometry (the B-side).

I am particularly interested in the relationship between mirror symmetry, deformation quantization (DQ), and geometric quantization (GQ). Gukov and Witten proposed an influential framework for quantizing symplectic manifolds using branes, often referred to as brane quantization. One consequence of their proposal suggests that the geometric quantization of a symplectic manifold M should admit an action from a holomorphic deformation quantization of a complexification of M. However, a rigorous mathematical definition of such an action had not been established. Together with my collaborators, we constructed the first concrete realization of "DQ acting on GQ" using Strominger–Yau–Zaslow (SYZ) mirror symmetry. Our construction also sheds light on defining the self-hom space of the so-called semi-affine canonical coisotropic A-brane.

At the same time, I am interested in constructing deformation quantizations on holomorphic symplectic manifolds arising from the Gross–Siebert program, as well as understanding their relationship with higher-genus Gromov–Witten invariants. I am also drawn to the open Gross–Siebert program, which aims to construct holomorphic vector bundles on Calabi–Yau manifolds via toric degenerations, tropical geometry, and (extended) scattering diagrams.

What are you interested in recently? Please share something about you, not math. (hobby, art, episode, travel story and etc.)

I have been playing the piano since middle school, and it remains one of my favorite hobbies. Recently, I purchased a high-quality electronic piano and plan to place it in my office so that I can relax during stressful moments. Music provides a different kind of structure and expression, and it helps me reset mentally when research becomes intense.

We'd like to hear about your dream and future plans. Do you have a role model or a philosophy of life?

At this stage of my career, my immediate goal is to obtain tenure. In many ways, I feel that I have already realized an important part of my dream—becoming a professor of mathematics. My philosophy is simple: "do what you do best". I believe in focusing on my strengths, refining them continuously, and contributing where I can be most effective.

Is there anything you want to tell to younger researchers?

Research can be frustrating, and progress is often slow. So first of all, do not give up too easily. At the same time, it is equally important to know when to step back. Persisting without reflection can lead to burnout. What could be more tragic than losing interest in the very subject that once inspired you? Take breaks. Talk to someone—about anything, not necessarily research. Take a nap. Enjoy a good meal. Small distractions can refresh your mind. Recharge yourself, and then continue the journey of exploration with renewed energy.

CGP Walk

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