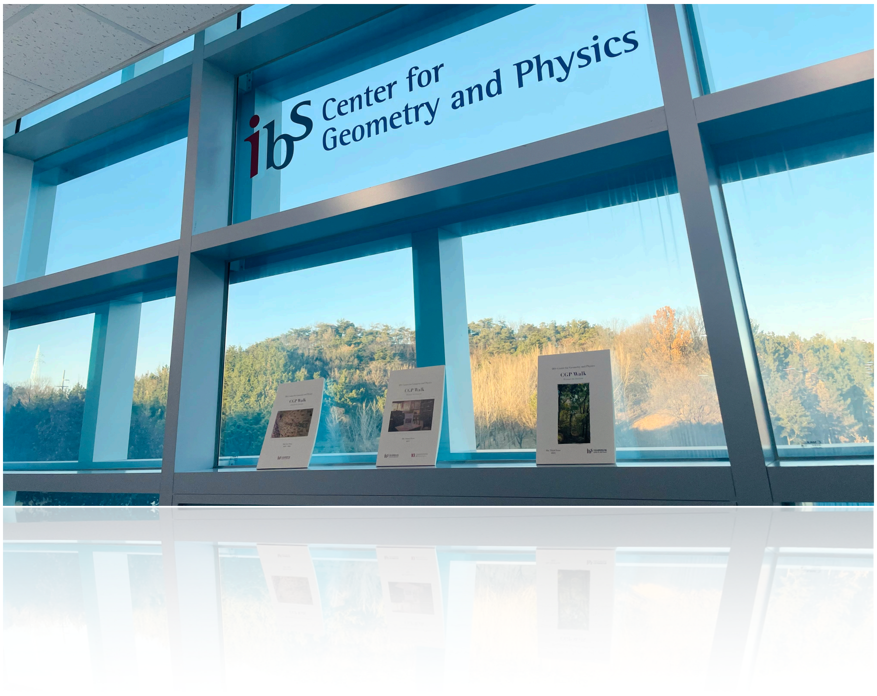
IBS Center for Geometry and Physics

CGP Walk

— Beyond the horizon —



The fourth Issue
2023

기초과학연구원
Institute for Basic Science

IBS Center for Geometry and Physics

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— *Beyond the horizon* —

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Institute for Basic Science

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Greetings

Director's Note

This year CGP had two important events, one the eleventh year onsite performance evaluation of CGP which is critical to the long-term presence of CGP itself, and the other the moving of CGP to a newly built IBS POSTECH campus.

With the collective effort of all CGP members, both research members and the administrative team, CGP has received the highest rating. I would like to take this opportunity to thank all CGP members for their hard work and devotion to the cause of CGP.

At last, after 10 years waiting, CGP has its own space in a nice quiet environment at a corner of POSTECH campus. I enjoy walking along the woods coming to the center and going home, and walking to the center after lunch along the woods enjoying daily sunlight and bird-watching. One of the participants in this year's workshops praised CGP's quiet environment by saying that he feels like being in the Oberwolfach Research Center for Mathematics[*]. He then added that it would have been even nicer if CGP had the luxury of having guest houses nearby and of having good catering service for the participants. I concur.

A few months ago, I had a chance to chat with my college friends who are also doing mathematics. They said that the youngsters in CGP, rather what they meant in Pohang, must have not known what they were doing when they came to the remote Pohang just pursuing their wholehearted scientific passion by leaving, some behind the busy culturally rich city life in Seoul and others behind their home country all over the world.

CGP's young researchers share common DNA of *dreamers*. History shows *what is already realized now* (sometime in the future) is *what they are dreaming about now*. We have a stark evidence of this in modern days: See mathematicians, scientists and engineers who worked in AT&T Bell Laboratory in the ages (1920–1980) of innovative advancement of modern communications[†]. I am thankful to and proud of working with these young dreamers.

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[*] The Oberwolfach Research Institute for Mathematics (German: Mathematisches Forschungsinstitut Oberwolfach) is a center for mathematical research in Oberwolfach, Germany. It was founded by mathematician Wilhelm Süss in 1944. It organizes weekly workshops on diverse topics where mathematicians and scientists from all over the world come to do collaborative research. The Institute is a member of the Leibniz Association, funded mainly by the German Federal Ministry of Education and Research and by the state of Baden-Württemberg. It also receives substantial funding from the Friends of Oberwolfach foundation, from the Oberwolfach Foundation and from numerous donors. – *from Wikipedia* –

[†] See the book 'The Idea Factory: Bell Labs and the Great Age of American Innovation', published in February 26, 2013.

Associate Director's Note

I joined the Department of Mathematics at POSTECH in May 2003. Now, it has been exactly 20 years. The significant change during my two decades at POSTECH occurred in 2012. In September 2012, I was dispatched from POSTECH to start a new journey at the IBS Center for Geometry and Physics. I vividly remember the hot summer of that year, filled with discussions and planning for the establishment of the center. Interviews for hiring administrative staff, awkward interactions with government officials, grappling with applications to hire researchers – whether the memories were positive or challenging, everything was filled with hope and expectation at that time. Yet, as time has passed, that vibrant day has receded into the distance, akin to a kite over the hill carried away by the wind. Ten years and one more have elapsed, and now I find myself standing in the profound valley of the Pohang Accelerator Laboratory.

In May 2023, IBS-CGP moved to a new building within the campus of the Pohang Accelerator Laboratory, leaving behind a dynamic decade on the POSTECH campus. There were many challenges along the way – the early days when everything was unfamiliar, the dark times of the pandemic, and the period of new beginnings and challenges brought about by the move. At the beginning I seemed to dream of idealistic visions, walking amidst floating white clouds. Now the idealistic blueprint has faded and I stand firmly with my feet on the ground. Looking out my new office window at the swaying trees, I suddenly feel that the past ten years have passed like the wind through the branches.

What have I done? I spent time in the center with interesting and sometimes painful problems, such as the problems of non-rationality of three-dimensional Fano varieties that I have admired since graduate school, the problems of special metrics on 5-dimensional Riemannian manifolds that I have deeply cherished, and the challenges of affine geometry that were brought about by a series of accidents. I do not claim that doing mathematics is entirely enjoyable work. It is a pilgrimage marked by pain and frustration. Over the past ten years, I believe I have shared this pilgrimage with IBS-CGP, experiencing the essence of mathematics. Although it involves pain and frustration, the essence of mathematics is freedom. Mathematics is like a gentle breeze that gently shakes the trees of my thoughts. I love to write poetry. I enjoy the process of nurturing the freedom of my emotions through the expression of language. Mathematics is the act of writing poetry with reason. From the beginning of IBS-CGP, I have been engaged in this free poetic process of reason.

Many mathematicians, or rather many connections, have passed through IBS-CGP. Counting all the mathematicians who have been through IBS-CGP in the past ten years would be challenging. Since I joined IBS-CGP, I have spent the past 10 years with the same heart as a mountain lodge manager who provides a breathing space for tired hikers on a rugged mountain. During this long period (if my memory serves me right), I encountered a total of 16 administrative and technical staff members belonging to IBS-CGP. They contributed significantly to the establishment of our unique mathematical research center. In particular, I would like to express deep gratitude to KIM Youngmi and SINN Elisa, who worked hard as team leaders for the IBS-CGP administration group, especially during the early days, to establish many things on the foundation that did not exist. Even now, I hear news of the 16 administrative and technical staff members through various channels. I am happy that they are all establishing their own positions in their respective fields.

As I reflect on the essential components of a mathematical research institute, I always emphasize that human resources are the most crucial element. From this perspective, I believe that IBS-CGP has had a successful ten years. Everyone filled the center with youthful and bright ideas, lifting it to where it stands today. Whether researchers or administrative and technical staff, everyone shared dreams and hopes, experiencing a decade filled with joy and frustration.

When I first entered IBC-CGP in 2012, I wrote Nikos Kazantzakis's epitaph on my office window:

> *I hope for nothing.*
> *I fear nothing.*
> *I am free.*

Now, as I embark on a new beginning, I find myself inclined to etch these words once again on my office window.

Background and Vision

The Center for Geometry and Physics (CGP) was founded in July 2012 as one of the first research centers of the Institute for Basic Science (IBS). The CGP originated in a government funded award, via IBS, to the research program of its director Yong-Geun Oh. This program aims to help establish and develop the emerging field of symplectic algebraic topology through a collaborative effort by experts in fields such as symplectic geometry, dynamical systems, algebraic geometry and mathematical physics. The Center is currently evolving into an international institution with a broader scope, focusing more generally on geometry and mathematical physics.

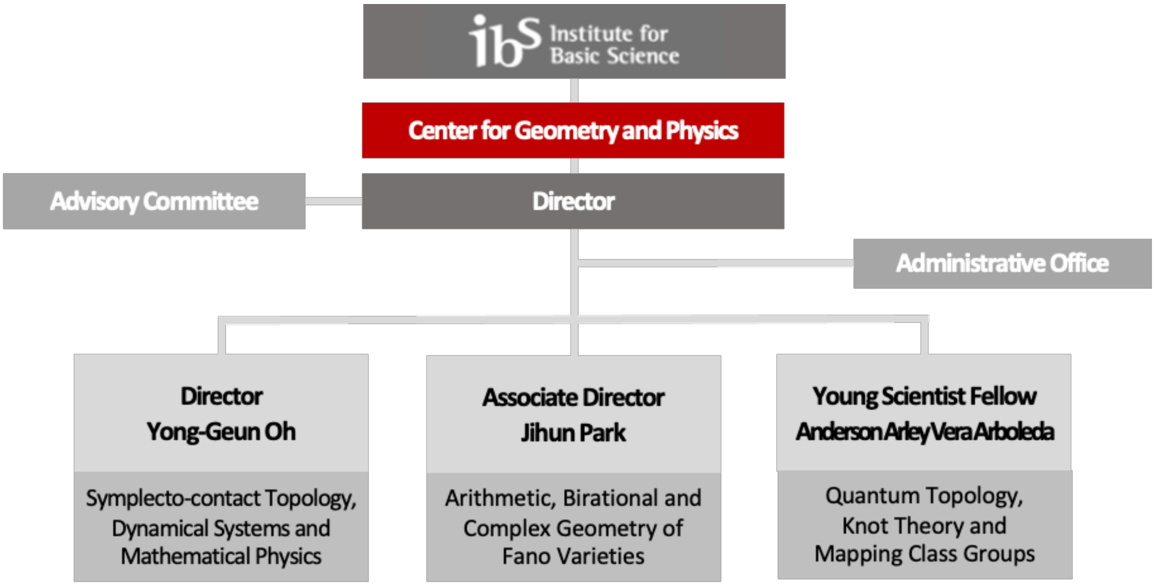
An ideal scientific research institute should be a place which fosters the disinterested pursuit of learning and the fundamental innovative thinking that advances the individual fields of inquiry of an intellectual community. The mission of the Center for Geometry and Physics (CGP) is to enable the research environment at the CGP to achieve this ideal. By now, the CGP has created such an atmosphere that ideas from mathematics and physics are naturally shared and interact. The CGP will ensure maintaining this unique research environment to serve the international community by

- Providing intellectual leadership and stewardship, guiding the development of relevant mathematics in fruitful directions by strengthening the interaction between geometry and physics,
- Playing the role of a physical nexus in Korea and beyond for the events which are the social glue of mathematical progress by hosting workshops, conferences, visitors, and so on, in order to lubricate the flow of ideas throughout the international community,
- Becoming as an incubator for young mathematicians, giving them the time and freedom to pursue ambitious and idiosyncratic research goals in a nurturant and enriching environment.

Organization

One arching research theme of the CGP is to promote interaction between symplectic geometry, algebraic geometry and mathematical physics in the study of symplectic topology and homological mirror symmetry and their applications to theoretical physics.

The CGP is organized into multi research groups, each of which comprises a senior scholar and several researchers whose areas of expertise and interest overlap synergistically.



Research Groups

Symplectic topology, Hamiltonian dynamics and mirror symmetry

Team Leader: Yong-Geun Oh

The current status of symplectic topology resembles that of classical topology in the middle of the twentieth century. Over time, a systematic algebraic language was developed to describe problems in classical topology. Similarly, a language for symplectic topology is emerging, but has yet to be fully developed. The development of this language is much more challenging both algebraically and analytically than in the case of classical topology. The relevant homological algebra of A_∞ structures is harder to implement in the geometric situation due to the analytical complications present in the study of pseudo-holomorphic curves or "instantons" in physical terms. Homological mirror symmetry concerns a certain duality between categories of symplectic manifolds and complex algebraic varieties. The symplectic side of the story involves an A_∞ category, called the Fukaya category, which is the categorified version of Lagrangian Floer homology theory. In the meantime, recent developments in the area of dynamical systems have revealed that the symplectic aspect of area preserving dynamics in two dimensions has the potential to further understanding of these systems in deep and important ways.

Research members and their research themes:

- **Sam Bardwell-Evans** (Symplectic geometry, Moduli spaces, Pseudoholomorphic curves and discs, Mirror symmetry)
- **Volker Genz** (Explicit problems in representation theory)
- **Jaekwan Jeon** (Deformations of rational surface singularities and related topics in symplectic geometry)
- **Jongmyeong Kim** (Homological mirror symmetry)
- **Norton Lee** (Supersymmetry, Integrable Systems, Quantum Field Theories, Mathematical Physics)
- **Seul Bee Lee** (Ergodic theory of dynamical systems, Number Theory and Geometric Group Theory)
- **Yong-Geun Oh** (Symplectic topology, Hamiltonian dynamics and mirror symmetry)

Arithmetic, Birational and Complex Geometry of Fano Varieties

Team Leader: Jihun Park

Fano varieties are algebraic varieties whose anticanonical classes are ample. They are classical and fundamental varieties that play many significant roles in contemporary geometry. Verified or expected geometric and algebraic properties of Fano varieties have attracted attentions from many geometers and physicists. In spite of extensive studies on Fano varieties for more than one centuries, numerous features of Fano varieties are still shrouded in a veil of mist. Contemporary geometry however requires more comprehensive understanding of Fano varieties.

Research members and their research themes:

- **Igor Krylov** (Birational geometry)
- **Jihun Park** (Birational geometry)

Mathematical Physics

Team Leader: Alexander Aleksandrov and Yong-Geun Oh

The mathematical relevance and deep interconnections between theoretical physics and mathematics are well-established. This subject is universally appreciated for its integrative role and for being one of the most fruitful sources of new ideas, theories and methods, and have numerous powerful applications to problems in mathematics, in particular, of geometry and topology. In recent decades, there have been various developments in supersymmetric quantum field theories and string/M-theory. In this premise, matrix models, integrable systems, Chern-Simons gauge theory, Landau-Ginzburg theory and mirror symmetry, and topological quantum field theories are the main themes of research pursued in this group.

Research members and their research themes:

- **Alexander Aleksandrov** (Mathematical physics, random matrix models, integrable systems, enumerative geometry)
- **Hisayoshi Muraki** (Noncommutative geometry, nongeometric backgrounds in supergravity, discretized geometry, matrix model)
- **Dmytro Voloshyn** (Mathematical physics, cluster algebras, Poisson geometry, quantum groups, integrable systems)

Quantum Topology, Knot Theory and Mapping Class Groups

Team Leader: Anderson Arley Vera Arboleda (Young Scientist Fellow)

Quantum topology lies in the intersection of algebra, topology and mathematical physics and it is a source of knot and 3-manifold invariants (quantum invariants) giving rise to the so-called Topological quantum field theories (TQFT). The construction of such TQFTs requires techniques from representation theory, combinatorics and topology. We mainly pursue two research directions:

(a) topological "meaning" of quantum invariants, this leads to the study of classical invariants such as Milnor invariants as well as the study of the mapping class groups and their representation theory.
(b) construction of new invariants for braids, knots and 3-manifolds, in particular, we point to a development of quantum invariants for knots in thickened surfaces.

Research members and their research themes:

- **Anderson Arley Vera Arboleda** (quantum topology, knot theory, mapping class groups)

CGP Advisory Committee

The CGP Advisory Committee consists of eight distinguished scholars from Korea and abroad. The committee meets once a year and provides advice and input on the operations of the Center.

The current members of the Advisory Committee are (as of December 2023):

Alexander Givental
Professor at University of California, Berkeley

Jae-Hun Jung
Professor at Pohang University of Science and Technology (POSTECH)

Mikhail Kapranov
Professor at Kavli IPMU, University of Tokyo

Ludmil Katzarkov
Professor at University of Miami & University of Vienna & Institute of Mathematics and Informatics

JongHae Keum
President at Korean Mathematical Society
Professor at Korea Institute for Advanced Study (KIAS)

Jongil Park
Professor at Seoul National University

Kyewon Koh Park
Emeritus Professor at Ajou University

Herman Verlinde
Professor at Princeton University

Research Infrastructure

The Center for Geometry and Physics aims to provide a research environment in which new and original ideas are boldly proposed, tested and revised by means of scientific interactions and communication. By doing so, we hope that some of those ideas evolve into a mature form of truly new mathematics. Thus, the goal of the center is to become the birthplace of fundamentally new research areas in addition to carrying out those projects envisioned in its initial proposal.

CGP Delta & Library

The CGP Delta is the central location of the Center's academic and social activities.

CGP Delta serves as the venue for the seminars, talks hosted by the Center. Members and visitors often gather here in small groups for discussions, exchange of ideas or simply for relaxing.

The CGP library collection of 4,576 books that the Center has established in topics related to the research areas of the Center greatly complements the rich archive sources available to its members and visitors. The entire CGP library collection is housed along the corridor of CGP Delta.

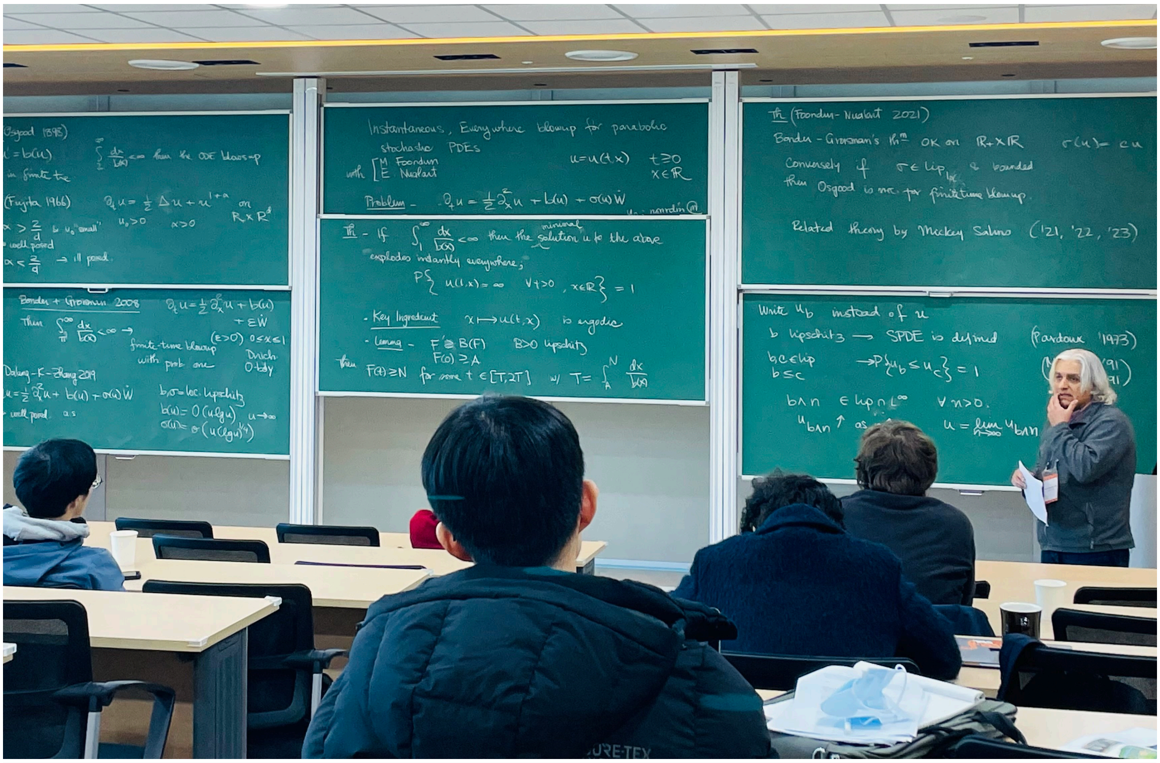
Website, Video System, and Other Facilities

The CGP website (http://cgp.ibs.re.kr) provides schedules and information on events hosted by the Center, and the preprints of the CGP members and the database for the entire collection of the CGP library which can be searched by title, author, ISBN, or year of publication.

In addition, video recordings of most talks, lectures, and conferences hosted by the CGP are uploaded and made available on the website with the consent of speakers. This feature allows anyone who is interested to access and benefit from the talks regardless of their physical location.

For the convenience of visitors and job applicants and the efficiency of the application process, the Center has implemented application features on its website. Those who are interested in visiting and conducting collaborative research with the members of the Center or who are interested in a research position at the Center can apply online at the website.

Scientific Activities



Statistics

- 6 conferences
- 14 colloquium talks and 46 seminar talks
- 2 lecture series

CGP at a Glance

JAN

Event
Launched: Eastern Hemisphere Colloquium on Geometry and Physics (EHCGP), Biweekly

FEB

MAR

APR

Event
Mini-workshop on Low-dimensional Topology (April 25 – 27)

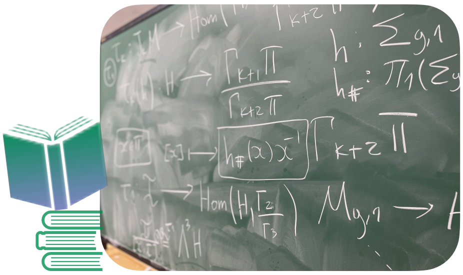
MAY

Event
Workshop on Moduli, K-stability, Fano varieties, and related topics (May 15 – 19)

News
Relocation in new building!
(IBS POSTECH Campus)

JUN

2023



2023

JUL

AUG

Event

CGP 11th Year On-Site Evaluation
(August 16 – 18)

SEP

New Research Member
Sam Bardwell-Evans

MOU
CRM, initiated for 3 years
(December 1)

OCT

Event

YeungNam Workshop on Algebraic Geometry XI
(October 13 – 14)

Fano varieties, their Geometry and Moduli
(October 30 – November 3)

NOV

New Research Member
Jaekwan Jeon

Event
Workshop in Kinetic theory,
thermodynamics and contact topology
(November 27 – December 1)

DEC

Event
Winter School on Low-dimensional Topology and Related Topics (December 11 – 15)

Conferences

In an effort to take on a leading role in enriching the mathematical society, the Center works in collaboration with other institutes and organizations to hold conferences for a wider audience of mathematicians and scholars. The Center has organized or co-organized 1 series of colloquium and 6 conferences:

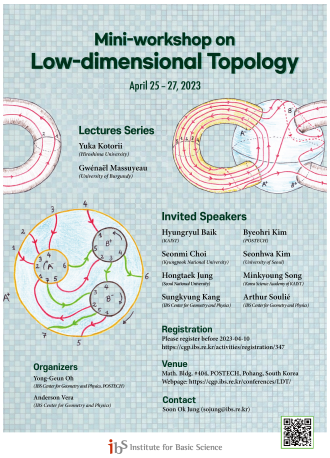
- **Eastern Hemisphere Colloquium on Geometry and Physics (EHCGP)**; since January 18, 2023 (biweekly)

> *The interplay of Geometry and Physics has traditionally been a mutually enriching process for both fields. Recent years have brought new remarkable discoveries and new points of view, connecting deep mathematics with important physical problems. It is an exciting time. The goal of our Colloquium is to promote the development and the mutual interaction of Geometry and Physics in the Asia-Pacific region. An international effort, it is conducted online to ensure the widest possible participation.*

- Organizing Committee: Sinya Aoki (YITP), Cheol-Hyun Cho (Seoul National University),
Mikhail Kapranov (Kavli IPMU), Xiaobo Liu (Beijing University & BICMR),
Paul Norbury (University of Melbourne), Yong-Geun Oh (IBS-CGP & POSTECH),
Kaoru Ono (RIMS), Gang Tian (Beijing University & BICMR),
Junwu Tu (ShanghaiTech University), Masahito Yamazaki (Kavli IPMU)

- Invited Speakers: Hirosi Ooguri (California Institute of Technology & Kavli IPMU)
Yongbin Ruan (IASM, Zhejiang University)
Dongmin Gang (Seoul National University)
Geordie Williamson (Sydney Mathematical Research Institute, University of Sydney)
Tadashi Takayanagi (YITP, Kyoto University)
Stavros Garoufalidis (Southern University of Science and Technology & MPIM)
Hiraku Nakajima (Kavli IPMU, University of Tokyo)
Zhen Huan (Center for Mathematical Sciences, HUST)
Nicolai Reshetikhin (YMSC, Tsinghua University & University of California - Berkeley)
Si Li (Tsinghua University)
Amihay Hanany (Imperical College London)

- **Mini-workshop on Low-dimensional Topology**; April 25 – 27, 2023
  - Organizers: Yong-Geun Oh (IBS-CGP & POSTECH)
    Anderson Vera (IBS-CGP)
  - Invited Lecturers: Yuka Kotorii (Hiroshima University)
    Gwénaël Massuyeau (University of Burgundy)
  - Invited Speakers: Hyungryul Baik (KAIST)
    Seonmi Choi (Kyungpook National University)
    Hongtaek Jung (Seoul National University)
    Sungkyung Kang (IBS-CGP)
    Byeohri Kim (POSTECH)
    Seonhwa Kim (University of Seoul)
    Minkyoung Song (Korea Science Academy of KAIST)
    Arthur Soulié (IBS-CGP)



Mini-workshop on
Low-dimensional Topology
April 25 – 27, 2023

Lectures Series
Yuka Kotorii
(Hiroshima University)
Gwénaël Massuyeau
(University of Burgundy)

Invited Speakers

Hyungryul Baik (KAIST)	Byeohri Kim (POSTECH)
Seonmi Choi (Kyungpook National University)	Seonhwa Kim (University of Seoul)
Hongtaek Jung (Seoul National University)	Minkyoung Song (Korea Science Academy of KAIST)
Sungkyung Kang (IBS Center for Geometry and Physics)	Arthur Soulié (IBS Center for Geometry and Physics)

Registration
Please register before 2023-04-10
https://cgp.ibs.re.kr/activities/registration/347

Venue
Math. Bldg. #404, POSTECH, Pohang, South Korea
Webpage: https://cgp.ibs.re.kr/conferences/LDT/

Organizers
Yong-Geun Oh
(IBS Center for Geometry and Physics, POSTECH)
Anderson Vera
(IBS Center for Geometry and Physics)

Contact
Soon Ok Jung (sojung@ibs.re.kr)

Institute for Basic Science

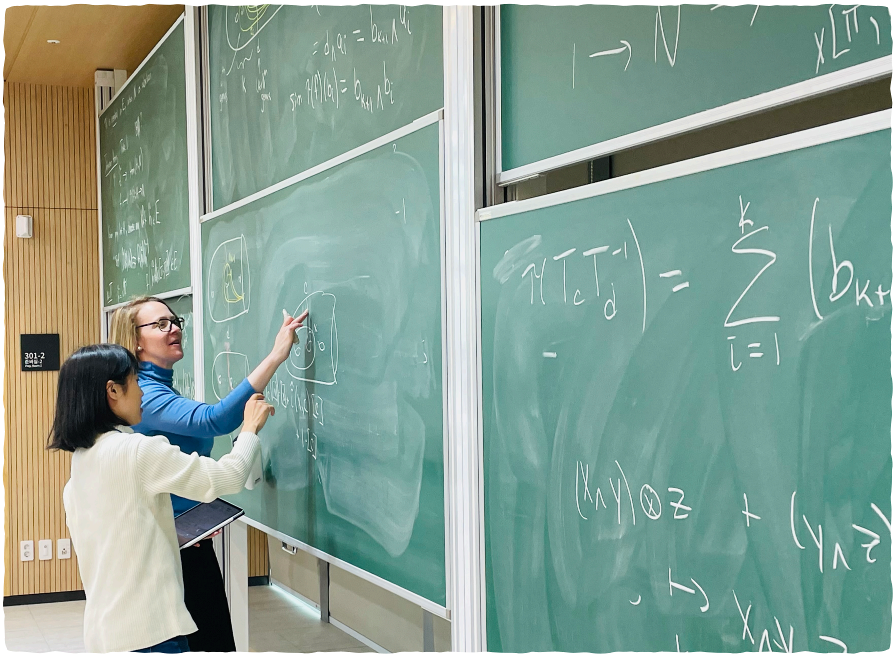
- **Workshop on Moduli, K-stability, Fano varieties, and related topics**; May 15 – 19, 2023
  - Organizers: Yongnam Lee (IBS Center For Complex Geometry, KAIST)
    Jihun Park (IBS-CGP & POSTECH)
  - Invited Speakers: Arnaud Beauville (University of Nice)
    Fabrizio Catanese (University of Bayreuth)
    Thibaut Delcroix (University of Montpellier)
    Kento Fujita (Osaka University)
    Young-Hoon Kiem (KIAS)
    Shigeru Mukai (RIMS, Kyoto University)
    Yuri Prokhorov (Steklov Mathematical Institute)
    Constantin Shramov (Higher School of Economics University)

- **Fano varieties, their Geometry and Moduli**; October 30 – November 3, 2023
  - Organizers: Hamid Abban (University of Nottingham)
    Livia Campo (KIAS)
    Young-Hoon Kiem (KIAS)
    Jihun Park (IBS-CGP & POSTECH)
    Joonyeong Won (Ewha Womans University)
  - Invited Speakers: Florin Ambro (Simion Stoilow)
    Kenny Ascher (University of California - Irvine)
    Paolo Cascini (Imperial College London)
    Ivan Cheltsov (University of Edinburgh)
    Kristin DeVleming (University of Massachusetts Amherst)
    Maksym Fedorchuk (Boston College)
    Kento Fujita (Osaka University)
    Masafumi Hattori (Kyoto University)
    Jun-Muk Hwang (IBS Center For Complex Geometry, KAIST)
    In-kyun Kim (Yonsei University)
    Igor Krylov (IBS-CGP)
    Yongnam Lee (IBS Center For Complex Geometry, KAIST)
    Takuzo Okada (Saga University)
    Karol Palka (Institute of Mathematics, Polish Academy of Sciences)
    Yuri Prokhorov (Steklov Mathematical Institute)
    Miles Reid (University of Warwick)
    Evgeny Shinder (University of Sheffield)
    Constantin Shramov (Higher School of Economics University)
    Sho Tanimoto (Nagoya University)
    Andrey Trepalin (Higher School of Economics University)
    Antonio Trusiani (University of Toulouse)
    Chuyu Zhou (National Taiwan University)

- **YeungNam Workshop on Algebraic Geometry XI**; October 13 – 14, 2023
  - Organizer: Jihun Park (IBS-CGP & POSTECH)
  - Invited Speakers: Youngook Choi (Yeungnam University)
    Eunjeong Lee (Chungbuk National University)
  - Participants: Seonja Kim (Chungwoon University)
    Yeongrak Kim (Pusan National University)
    Hosung Kim (Changwon National University)
    Kyeong-Dong Park (Gyeongsang National University)
    Kyoung-Seog Lee (POSTECH)
    Wanseok Lee (Pukyong National University)
    Junmyeong Jang (University of Ulsan)
    Kiryong Chung (Kyungpook National University)
    Kangjin Han (DGIST)

- **Workshop in Kinetic theory, thermodynamics and contact topology**; November 27 – December 1, 2023
  - Organizer: Jin Woo Jang (POSTECH)
    Kunwoo Kim (POSTECH)
    Yong-Geun Oh (IBS-CGP & POSTECH)
  - Invited Speakers: Young-Pil Choi (Yonsei University)
    Jacky Chong (Peking University)
    Tadahisa Funaki (BIMSA & University of Tokyo)
    Hao Ge (BICMR & Peking University)
    Shin-itiro Goto (Chubu University)
    Jin Woo Jang (POSTECH)
    Jinwook Jung (Jeonbuk National University)
    Davar Khoshnevisan (University of Utah)
    Chanwoo Kim (University of Wisconsin-Madison)
    Jaelin Kim (Alfréd Rényi Institute of Mathematics)
    Jinsu Kim (POSTECH)
    Laurent Lafleche (École normale supérieure de Lyon)
    Ji Oon Lee (KAIST)
    Kyeongsik Nam (KAIST)
    Yong-Geun Oh (IBS-CGP & POSTECH)

- **Winter School on Low-dimensional Topology and Related Topics**; December 11 – 15, 2023
  - Organizer: Yong-Geun Oh (IBS-CGP & POSTECH)
    Anderson Vera (IBS-CGP)
  - Invited Lecturers: Tara Brendle (University of Glasgow)
    Zsuzsanna Dancso (University of Sydney)
    Jun Murakami (Waseda University)
    Jessica S. Purcell (Monash University)



Seminars

The CGP hosts various seminars given both by visiting scholars and the members of the Center.

Symplectic Monday Seminar (Mondays 16:00 – 18:00)[*]
The talks are focused on symplectic geometry and chaired by Director Yong-Geun Oh.

Algebraic Geometry Seminar (Tuesdays 16:00 – 18:00)[*]
The talks are focused on algebraic geometry and chaired by Associate Director Jihun Park.

Wednesday Noon Seminar (Wednesdays 12:00 – 13:00)
The Wednesday Noon Seminar runs weekly with lunch for talks by CGP members on various topics of their own research interest or current works. This is kind of semi-closed seminar open to CGP members and visitors only. *(Paused due to COVID-19)*

Director's Seminar (Bi-weekly Wednesdays 13:30 – 15:30)
The purpose of this seminar is to give updates on current developments and mathematical research highlights in general to CGP members and visitors, and to promote deeper interaction between the speaker and the audience. The seminar's general spirit reflects that of the famous Gelfand Seminar.

The Center for Geometry and Physics Seminar (Thursdays 16:00 – 18:00)
The Center for Geometry and Physics Seminar on every Thursday afternoon is the most important regular event of the CGP, and generally all members of the Center participate. The seminars are formatted to encourage robust and dynamic interactions among participants. The seminar is structured as a two-hour talk by a designated speaker with a thirty minute intermission with tea and snack. The first half is intended to be a colloquium-level talk suitable for a general mathematical audience, while the second half can be more specialized. Discussions may continue over dinner.

Mathematical Physics Seminar (Fridays 13:00 – 15:00)[*]
The talks are focused on mathematical physics chaired by Director Yong-Geun Oh and a research fellow, Alexander Aleksandrov.

IBS-CGP Post-doc Lecture Series
The Director encourages CGP post-doc members to give 3-4 one-hour lectures on their research area. The main purpose of the series is to train post-doc’s lecture skills so the Director comments on the lecture series and give suggestions to improve.

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[*] Depending on the invited speaker’s location, the online seminar was held flexibly at 10:00 – 11:00 or 16:00 – 18:00.

List of All Talks

Monopole Floer homology and holomorphic curves
Yi-Jen Lee (The Chinese University of Hong Kong)
December 18, 2023

DM stacks from surfaces and local Calabi-Yau 3-folds
Sung Woo Nam (POSTECH)
December 6, 2023

Reading seminar on symplectic and birational geometry
Yong-Geun Oh (IBS-CGP & POSTECH)
December 6, 2023

Augmentation Varieties and Disk Potential
Soham Chanda (Rutgers University)
December 4, 2023

Unimodular random rooted manifolds
Jaelin Kim (Alfréd Rényi Institute of Mathematics)
November 23, 2023

Automorphism groups of affine varieties with at most one $\mathbb{G}_a$-action
Alexander Perepechko (HSE University)
November 22, 2023

[IBS-CGP&POSTECH-Math Colloquium]
Low Dimensional Topology and Algebraic Geometry
Kyoung-Seog Lee (POSTECH)
November 17, 2023

[IBS-CGP Colloquium]
Towards Quantum Computing with Spins on Surfaces
Andreas Heinrich (IBS Center for Quantum Nanoscience, Ewha Womans University)
November 16, 2023

Toric vector bundles, non-abelianization and spectral networks
Yat-Hin Suen (KIAS)
November 13, 2023

Diophantine approximation in the view point of homogenous dynamics and its S-arithmetic generalization
Jiyoung Han (KIAS)
November 9, 2023

Birational Models of Fano hypersurfaces and K-stability
Tiago Duarte Guerreiro (University of Essex)
November 7, 2023

Floer cohomology of compositions of Lagrangian Dehn twists
Weiwei Wu (Zhejiang University)
November 6, 2023

Diagram genus, generators and applications
Alexander Stoimenow (Dongguk University WISE)
October 25, 2023

Sarkisov links of projective 3-space initiated by a divisorial contraction with centre a point
Erik Paemurru (Saarland University)
October 24, 2023

Holomorphic curves, dynamics and the three-body problem
Otto van Koert (Seoul National University)
October 23, 2023

[IBS-CGP&POSTECH-Math Colloquium]
Applications of automorphic forms
Sug Woo Shin (University of California - Berkeley)
October 20, 2023

[IBS-CGP Post-doc Lecture Series]
On triangulated persistence categories I – III
Jongmyeong Kim (IBS-CGP)
October 17 – 19, 2023

Introduction to (Relative) Symplectic Cohomology and Applications
Dahye Cho (Yonsei University)
October 16, 2023

Symmetry vs. Transversality for the moduli space of bordered stable maps
Kenji Fukaya (Simons Center for Geometry and Physics)
October 5, 2023

[EHCGP]
Computational Methods for Symplectic Singularities
Amihay Hanany (Imperial College London)
October 4, 2023

[EHCGP]
Elliptic Chiral Index, HKR and Holomorphic Anomaly
Si Li (Tsinghua University)
September 13, 2023

Induced Morphisms of Moduli Spaces of Pseudoholomorphic Discs
Sam Bardwell-Evans (IBS-CGP)
September 11, 2023

Tropical Coamoeba, Calabi-Yau Mirror Symmetry and the Combinatorics of Dimers
Rak-Kyeong Seong (UNIST)
September 8, 2023

[Intensive Lecture Series]
Frobenius splitting and wonderful compactifications I – III
Michel Brion (Institut Fourier)
August 29 – 31, 2023

Automorphism groups of del Pezzo surfaces
Aurore Boitrel (Université d'Angers)
August 30, 2023

Non-maximality of algebraic subgroups of Cremona groups
Susanna Zimmermann (University of Paris - Saclay)
August 29, 2023

Calabi–Yau structures on Rabinowitz Fukaya categories
Jongmyeong Kim (IBS-CGP)
August 3, 2023

Family Floer theory, non-abelianization and Spectral Networks
Yoon Jae Nho (University of Cambridge)
July 31, 2023

Classical, quantum, and isomonodromic Seiberg-Witten geometry of A-type theories
Nikita Nekrasov (Simons Center for Geometry and Physics)
July 26, 2023

Asymptotic nonvanishing of syzygies of algebraic varieties
Jinhyung Park (KAIST)
July 25, 2023

Exploring the cohomology of a regular semisimple Hessenberg variety
Eunjeong Lee (Chungbuk National University)
July 11, 2023

Cylindrical ample divisors on Du Val del Pezzo surfaces
Masatomo Sawahara (Yokohama National University & Saitama University)
June 27, 2023

Maximally non-factorial Fano varieties
Igor Krylov (IBS-CGP)
June 13, 2023

Minimal rational curves on complete symmetric varieties
Shinyoung Kim (IBS-CGP)
May 23, 2023

[EHCGP]
On the statistics of indecomposable components in large tensor products of representations Lie algebras and quantum groups
Nicolai Reshetikhin (YMSC, Tsinghua University & University of California - Berkeley)
May 17, 2023

Quantum Entanglement and related topics
Yong-Geun Oh (IBS-CGP & POSTECH)
May 17, 2023

Rationality problem for conic bundles
Yuri Prokhorov (Steklov Mathematical Institute)
May 12, 2023

Birational geometry of del Pezzo surfaces
Constantin Shramov (HSE University)
May 9, 2023

[EHCGP]
Twisted Real quasi-elliptic cohomology
Zhen Huan (Center for Mathematical Sciences, HUST)
May 3, 2023

Quantum Entanglement and related topics
Norton Lee (IBS-CGP)
April 28, 2023

Equivariant Structures in Symplectic Floer Homotopy
Semon Rezchikov (Princeton University)
April 24, 2023

Operator algebras in AdS/CFT: bulk reconstruction, quantum extremal surfaces, and baby universe
Jinwoo Kang (California Institute of Technology)
April 20, 2023

[EHCGP]
Orthosymplectic bow varieties
Hiraku Nakajima (Kavli IPMU, University of Tokyo)
April 19, 2023

Quantum Entanglement and related topics
Norton Lee (IBS-CGP)
April 19, 2023

A universal formula for the density of states with global symmetry
Jinwoo Kang (California Institute of Technology)
April 17, 2023

Wrapped Fukaya category of plumbings of cotangent bundles of spheres
Dogancan Karabas (Kavli IPMU, University of Tokyo)
April 13, 2023

Quantum Entanglement and related topics
Hisayoshi Muraki (IBS-CGP)
April 12, 2023

Fano manifolds with big tangent bundles
Jeong-Seop Kim (KIAS)
April 11, 2023

[EHCGP]
What is a holomorphic quantum modular form?
Stavros Garoufalidis (Southern University of Science and Technology & MPIM)
April 5, 2023

Flags on Fano 3-fold hypersurfaces
Livia Campo (KIAS)
April 4, 2023

Simplicial decompositions of Weinstein sectors and tangle contact homology
Johan Asplund (Columbia University)
March 30, 2023

Homological Mirror Symmetry of Degenerate Cusp Singularities and their Representations
Kyungmin Rho (Kyungmin Rho)
March 27, 2023

[EHCGP]
Holography with End-of-the-World Branes and Quantum Entanglement
Tadashi Takayanagi (Yukawa Institute for Theoretical Physics, Kyoto University)
March 22, 2023

Bounds of weighted complete intersections and their Hodge numbers
Victor Przyjalkowski (Steklov Mathematical Institute)
March 14, 2023

[EHCGP]
What can the working (pure) mathematician expect from deep learning?
Geordie Williamson (Sydney Mathematical Research Institute, University of Sydney)
March 8, 2023

Translated points and Contact dynamical systems
Dylan Cant (Univesity of Montreal)
March 6, 2023

ACC of plc thresholds
Sung Rak Choi (Yonsei University)
February 24, 2023

[EHCGP]
Infra-red phases of class R theories
Dongmin Gang (Seoul National University)
February 15, 2023

The Operad of Series Parallel posets and an identity of Ramanujan
Eric Rubiel Dolores Cuenca (Yonsei University)
February 2, 2023

[EHCGP]
Castelnuovo Bound and Higher Genus Gromov-Witten Invariants of Quintic 3-fold
Yongbin Ruan (Institute for Advanced Study in Mathematics, Zhejiang University)
February 1, 2023

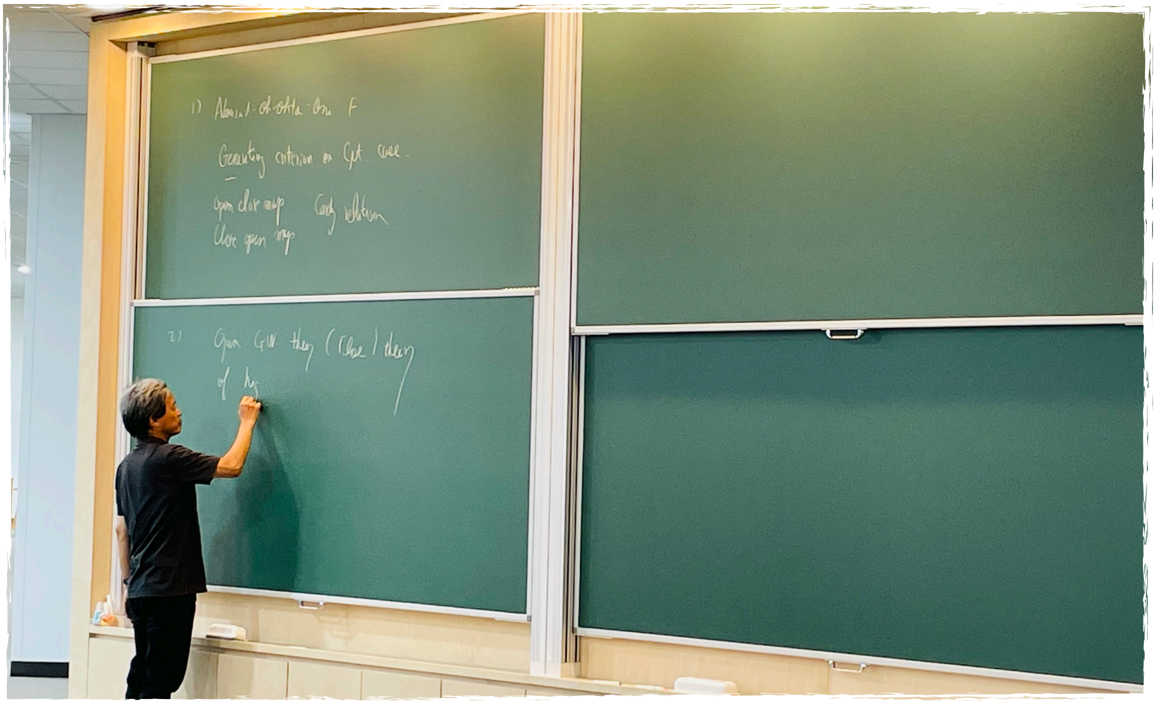
[EHCGP]
Symmetry-Resolved Density of States
Hirosi Ooguri (California Institute of Technology & Kavli IPMU)
January 18, 2023

Deviation spectrum of Birkhoff integrals for locally Hamiltonian flows on compact surfaces
Minsung Kim (Scuola Normale Superiore - Pisa)
January 5, 2023

Visitor Programs and Visitors

CGP runs programs to support visiting scholars. The goal of the visitor programs at the center is to support dynamic researchers working in topics related to the core fields of interest. In particular, the center aims to facilitate the active creation of new research and the dissemination of recent progress at the boundary of what is known. More concretely, we believe that mixing interesting people working on interesting problems in one place has the potential to reveal commonalities, promote collaboration, and help those people advance in understanding.

The center can provide office space and housing for approved visitors. Limited funds are available to support for travel and local expenses for visiting scholars.



List of All Visitors

Eunjeong Lee (Chungbuk National University)
December 19, 2023 – March 1, 2024
October 13 – 14, 2023
June 15 - August 31, 2023

Yi-Jen Lee (The Chinese University of Hong Kong)
December 12 – 19, 2023

Sung Woo Nam (POSTECH)
December 6, 2023

Boris Bychkov (University of Haifa)
November 26 – December 2, 2023

Sergei Shadrin (University of Amsterdam)
November 26 – December 2, 2023

Petr Dunin-Barkowski (HSE University)
November 26 – December 6, 2023

Maxim Kazarian (HSE University)
November 26 – December 3, 2023

Alexander Perepechko (HSE University)
November 19 – 25, 2023

Jaelin Kim (Alfréd Rényi Institute of Mathematics)
November 19 – December 1, 2023

Kyoung-Seog Lee (POSTECH)
November 17, 2023

Andreas Heinrich (IBS Center for Quantum Nanoscience, Ewha Womans University)
November 16, 2023

Yat-Hin Suen (KIAS)
November 12 – 18, 2023

Jiyoung Han (KIAS)
November 9 – 11, 2023

Tiago Duarte Guerreiro (University of Essex)
November 4 – 10, 2023

Alexander Stoimenow (Dongguk University WISE)
October 24, 2023

Otto van Koert (Seoul National University)
October 23 – 25, 2023

Sug Woo Shin (University of California - Berkeley)
October 20, 2023

Dahye Cho (Yonsei University)
October 16 – 17, 2023

Erik Paemurru (Saarland University)
October 13 – November 9, 2023

Seonja Kim (Chungwoon University)
October 13 – 14, 2023

Youngook Choi (Yeungnam University)
October 13 – 14, 2023

Kaoru Ono (RIMS, Kyoto University)
October 4 – 7, 2023

Hiroshi Ohta (Nagoya University)
October 4 – 7, 2023

Kenji Fukaya (Simons Center for Geometry and Physics)
October 4 – 7, 2023

Rak-Kyeong Seong (UNIST)
September 7 – 8, 2023

Kyeong-Dong Park (Gyeongsang National University)
August 29 – 31, 2023
January 16 – 19, 2023

Yongnam Lee (IBS Center for Complex Geometry)
August 29 – 31, 2023

Dongseon Hwang (IBS Center for Complex Geometry)
August 29 – September 1, 2023

Jaehyun Hong (IBS Center for Complex Geometry)
August 28 – September 1, 2023

Susanna Zimmermann (University of Paris-Saclay)
August 27 – 31, 2023

Aurore Boitrel (Université d'Angers)
August 27 – September 8, 2023

Michel Brion (Institut Fourier)
August 26 – September 1, 2023

Yoon Jae Nho (University of Cambridge)
August 21 – 24, 2023
July 31, 2023

Jaehyun Kim (Ewha Womans University)
August 20 – 25, 2023

Jinhyung Park (KAIST)
July 25 – 26, 2023

Nikita Nekrasov (Simons Center for Geometry and Physics)
July 24 – 27, 2023

Stefano Marmi (Scuola Normale Superiore)
June 29 – July 1, 2023

Dong Han Kim (Dongguk University)
June 29 – July 1, 2023

Masatomo Sawahara (Yokohama National University & Saitama University)
June 26 – July 1, 2023

Takashi Kishimoto (Saitama University)
June 26 – July 1, 2023

Constantin Shramov (HSE University)
May 7 – 20, 2023

Yuri Prokhorov (Steklov Mathematical Institute)
May 7 – 20, 2023

Jinwoo Kang (California Institute of Technology)
April 15 – 30, 2023

Jeong-Seop Kim (KIAS)
April 11 – 12, 2023

Hanwool Bae (Seoul National University)
April 11 – 15, 2023

Dogancan Karabas (Kavli IPMU, University of Tokyo)
April 9 – 15, 2023

Livia Campo (KIAS)
April 3 – 5, 2023

Kyungmin Rho (Seoul National University)
March 26 – 29, 2023

Johan Asplund (Columbia University)
March 26 – April 7, 2023

Victor Przyjalkowski (Steklov Mathematical Institute)
March 12 – 18, 2023

Kento Fujita (Osaka University)
March 12 – 18, 2023

Ivan Cheltsov (University of Edinburgh)
March 12 – 18, 2023

Sung Rak Choi (Yonsei University)
February 23 – 24, 2023

Wonbo Jeong (Seoul National University)
February 14 – 17, 2023

Hanwool Bae (Seoul National University)
February 14 – 17, 2023

Eric Rubiel Dolores Cuenca (Yonsei University)
January 29 – February 4, 2023

Minsung Kim (Scuola Normale Superiore - Pisa)
January 4 – 6, 2023

MOUs

The CGP has signed several MOUs for active research collaborations and academic exchanges with the mathematics community.

Beijing International Center for Mathematical Research (BICMR), China

November 2015 – November 2026 (renewed in 2021)

- 3rd BICMR&IBS-CGP Joint Symplectic Geometry Workshop (September 24 – 26, 2019 @ POSTECH)
- Silk Road Geometry Conference 2018 (June 4 – 8, 2018 @ Gökova Geometry/Topology Institute)
- 2nd BICMR&IBS-CGP Joint Symplectic Geometry Workshop (September 18 – 22, 2017 @ BICMR)
- 1st BICMR&IBS-CGP Joint Symplectic Geometry Workshop (October 31 – November 4, 2016 @ KAL Hotel, Jeju)

Research Institute for Mathematical Sciences (RIMS), Japan

August 2017 – July 2025 (renewed in 2020)

- 2021 Pacific Rim Complex & Symplectic Geometry Conference (July 12 – 16, 2021 | Online)
- RIMS & IBS-CGP Joint Symplectic Geometry Workshop (December 2 – 4, 2019 @ Kyoto University)
- Wall-crossing Formula, Open Gromov-Witten Invariants and Related Areas (October 29 – 31, 2018 @ POSTECH)
- Pacific Rim Complex-Symplectic Geometry Conference (July 31 – August 4, 2017 @ POSTECH)

Mathematical Research Institute (MATRIX), Australia

December 2018 – November 2026 (renewed in 2021)

- IBS-CGP and MATRIX workshop on Symplectic Topology (December 5 – 16, 2022 @ University of Melbourne[*])

The Centre de recherches mathématiques (CRM), Canada

December 2023 – December 2026

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[*] Creswick Campus

Research Highlights

Sungkyung Kang : Rationally slice knots in the smooth knot concordance group

Jongmyeong Kim : Towards a categorical interpretation of holomorphicity

Shin-young Kim : Greatest Ricci lower bounds of projective horospherical manifolds of Picard number one

Norton Lee : Quantum Hall effect

Dmitriy Voloshyn : Cluster algebras and Poisson Geometry

Rationally slice knots in the smooth knot concordance group

Sungkyung Kang
Titchmarsh Research Fellow, Mathematical Institute, University of Oxford
(CGP Research member from August 2020 to August 2023)

Introduction

My primary research interests lie in the area of low-dimensional topology, which is a study of topological objects in dimension at most four. Among various low-dimensional topological objects, the ones that I am mainly interested in are those in dimension three and four, and more importantly, the interplay between them. They include knots, links, 3-manifolds, 4-manifolds, and embedded surfaces in 4-manifolds.

One of the most important phenomena that occurs for those objects is *exoticness*, which refers to subtle differences between *topological* objects and *smooth* objects. Exoticness was first discovered in the high-dimensional realm; the first discovery of an exotic structure were the exotic 7-spheres by Milnor [Mil56]. However, exoticness in four dimensions is much more subtle than in higher dimensions, because while exotic structures in higher-dimensional manifolds can usually be completely classified using homotopy theoretic methods, the same classification problem for 4-manifolds remains largely unknown. Actually, unlike in higher dimensions, if a 4-manifold admits an exotic structure, then it usually admits infinitely many exotic structures, and there is no known example where this observation is proven to be false.

Exoticness in 4-manifolds, and surfaces embedded in them, are usually distinguished using gauge-theoretic methods, while Freedman's results [Fre82] are used to ensure topological equivalence; probably the most widely used one is Seiberg–Witten theory. However gauge-theoretic invariants are notoriously hard to compute except in simple cases like Brieskorn spheres and torus knots. This problem was partially resolved by the discovery of Heegaard Floer theory by Ozsváth and Szábo [OS04], which is conjecturally equivalent to Seiberg –Witten theory (proven in the case of 3-manifolds [KLT20]), but much easier to compute due to the presence of knot Floer homology [OS03, Ras03], surgery formulae [OS08], and bordered techniques [LOT18]. Heegaard Floer theory was thus used during the previous two decades to solve a lot of complicated exoticness problems.

However, while Seiberg–Witten theory is much less computable in general than Heegaard Floer theory, it has a clear advantage that its construction is much more straightforward and thus extra algebraic structures on it can be defined in a relatively easier way. For example, the $\mathrm{Pin}(2)$-symmetry of Seiberg–Witten equations was exploited by Manolescu [Man16] to define equivariant Seiberg–Witten Floer homology; he used it to disprove the triangulation conjecture in dimensions at least five. Such techniques are currently unavailable on the Heegaard Floer side.

In recent years, there were efforts to remedy this situation. Although the Heegaard Floer counterpart of $\mathrm{Pin}(2)$-equivariant Seiberg–Witten Floer homology is still not known, one can focus on the cyclic subgroup $\mathbb{Z}_4 \subset \mathrm{Pin}(2)$ generated by j. In this case, a conjectural counterpart, called involutive Heegaard Floer theory, was constructed by Hendricks and Manolescu [HM17], and further studied in a series of papers, including

[HHSZ20]. A similar strategy, with j replaced by the deck transformation action when the given 3-manifold is a branched double cover, was initiated in my work [AKS20] with Alfieri and Stipsicz, and developed in several papers, including [DMS23]. A lot of progress was made using these techniques, mainly by exploiting their computability. In this article, we will focus on how the involutive Heegaard Floer technique was applied to study the behavior of rationally slice knots in the smooth knot concordance group.

Knots, sliceness, and the concordance group

A *knot* is a continuously embedded circle in the 3-sphere S^3. In knot theory, we consider knots up to *isotopy*, i.e. two knots are isotopic if there is a continuous 1-parameter family of knots from one to another. The simplest example of knots is the *unknot*, which are the knots that can be "contracted to a point"; this is equivalent to saying that they bound disks in S^3.

Things get much more interesting when we start to deal with four dimensions, instead of three. We consider S^3 as the boundary of the standard smooth 4-ball B^4, and ask whether a given knot K, which lives in S^3, bounds a (continuously embedded) disk, say D, in B^4; if it does, we say that K is *slice*. However every knot bounds a disk in B^4, as we can just take a cone over K. Interestingly, this problem can be resolved in two different ways.

- We can require D to be *locally flat*, i.e. the disk D locally looks like the standard plane $\{0\} \times \mathbb{R}^2 \subset \mathbb{R}^2 \times \mathbb{R}^2$ at every point in the interior of D;
- We can require D to be smooth.

These two methods lead to totally different notion of sliceness; if we require D to be locally flat, we say that K is *topologically slice*, whereas if we require D to be smooth, we say that K is *smoothly slice*.

The notion of sliceness naturally induces two versions of *concordances*, which are equivalence relations. Given two knots K_1 and K_2, we say that they are *topologically concordant* if $K_1 \sharp -K_2$ is topologically slice, and *smoothly concordant* if $K_1 \sharp -K_2$ is smoothly slice. Since $K \sharp -K$ is smoothly (and thus also topologically) slice, the concordance classes of knots form abelian groups under connected sum operation. If we take topological concordance classes, we call the resulting group $\mathscr{C}_T$ the *topological concordance group*; if we take smooth concordance classes, we call the resulting group $\mathscr{C}$ the *smooth concordance group*.

We can also replace B^4 by any closed connected smooth 4-manifold X, simply by creating a puncture on it and identify the boundary of the punctured manifold with S^3 in which knots are defined. If a knot K bounds a smoothly embedded disk in the punctured X, we say that K is smoothly slice in X. An interesting case arises when we use punctured 4-manifolds which are *rational homology 4-balls*, i.e. whose $\mathbb{Q}$-coefficient singular homology are isomorphic to $H_*(B^4;\mathbb{Q})$. This setting defines the notion of rational sliceness; we say that a knot is *rationally slice* if it is smoothly slice in some rational homology 4-ball. This notion also defines an abelian group in the same way as how the smooth concordance group $\mathscr{C}$ is defined. We denote the resulting group $\mathscr{C}_{\mathbb{Q}}$ as the *smooth rational concordance group*.

From the above definitions, it is clear that we have the following naturally defined surjective "forgetful" maps:

$$\phi : \mathscr{C} \to \mathscr{C}_T, \quad \psi : \mathscr{C} \to \mathscr{C}_{\mathbb{Q}}.$$

Endo [End95] showed that $\ker \phi$ contains a $\mathbb{Z}^\infty$ subgroup, and Cha [Cha07] showed that $\ker \psi$ contains a $\mathbb{Z}_2^\infty$ subgroup. Furthermore, It was shown in [HKL16] that $\ker \psi \cap \ker \phi$ also contains a $\mathbb{Z}_2^\infty$ subgroup. However the torsion-free part of $\ker \psi$ was poorly understood; it was unknown whether $\ker \psi$ itself is a torsion group.

The involutive techniques

Given a knot K, the Heegaard Floer homology package associates to it a $\mathbb{Z}^2$-bigraded homotopy equivalence class of chain complexes $CFK_{UV}(S^3, K)$, which is finitely generated and free over the base ring $\mathbb{F}_2[U, V]$. Furthermore, given a smooth oriented decorated cobordism C between knots K_1 and K_2, which is basically an oriented and smoothly embedded surface with ends K_1 and K_2 (with extra data which we will neglect), in a twice-punctured Spin^c rational homology 4-sphere $(W, \mathfrak{s})$, the theory associates to it a bidegree-preserving chain map

$$F_{W,C,\mathfrak{s}} : CFK_{UV}(S^3, K_1) \to CFK_{UV}(S^3, K_2)$$

such that the localized map

$$(U, V)^{-1} F_{W,C,\mathfrak{s}} : (U, V)^{-1} CFK_{UV}(S^3, K_1) \to (U, V)^{-1} CFK_{UV}(S^3, K_2)$$

is a homotopy equivalence. In other words, if K_1 and K_2 are smoothly concordant, then there exist bidegree-preserving chain maps

$$F_1 : CFK_{UV}(S^3, K_1) \to CFK_{UV}(S^3, K_2),$$
$$F_2 : CFK_{UV}(S^3, K_2) \to CFK_{UV}(S^3, K_1),$$

such that localizing them by $(U, V)^{-1}$ induce homotopy equivalences. This condition is often called as *local equivalence*; we are saying that knot Floer chain complexes of smoothly concordant knots are locally equivalent. This is a very effective way of obstructing a knot from being slice.

The problem of this approach is that there are no further restriction for W, apart from being a rational homology 4-sphere. Since we are trying to detect subtle differences between sliceness and rational sliceness, this is a huge problem. One way to remedy this situation is to consider the conjugation symmetry

$$\iota_K : CFK_{UV}(S^3, K) \to CFKUV(S^3, K),$$

which is a homotopy skew-autoequivalence of $CFK_{UV}(S^3, K)$, defined for any knot K up to skew-homotopy. It is shown in [Zem19a] that if two knots K_1, K_2 are smoothly concordant (by a concordance C) in a spin rational homology ball $(W, \mathfrak{s})$, the chain map $F_{W,C,\mathfrak{s}}$ homotopy-commutes with ι_K, i.e.

$$\iota_{K_2} \circ F_{W,C,\mathfrak{s}} \text{ is skew-homotopic to } F_{W,C,\mathfrak{s}} \circ \iota_{K_1}.$$

Since we now require W to be spin, we can detect the difference between sliceness and rational sliceness. To be more precise, if two knots are smoothly concordant, then their knot Floer chain complexes are locally equivalent via maps which homotopy-commute with ι_K; this condition is called as ι_K-*local equivalence*. In fact, it was already discovered in [HM17] that the knot Floer chain complex of the figure-eight knot 4_1 and the unknot U are locally equivalent (which agrees with the fact that it is rationally slice) but not ι_K-locally equivalent.

Results

To apply the involutive techniques, we need a way to compute the action of ι_K on complicated knots. One possible way is to use the fact that it squares to the Sarkar involution:

$$\iota_K^2 \sim \xi_K \sim 1 + \Phi_K \Psi_K,$$

where Φ_K and Ψ_K are the formal derivatives of the differential of $CFK_{UV}(S^3, K)$ with respect to the formal variables U and V [Zem19b]. For simple knots such as Floer-thin knots and L-space knots, it is shown in [HM17] that this condition uniquely determines ι_K up to basis change. Fortunately, this was also the case for the cabled knots

$$K_n = (4_1)_{2n+1,1}, \quad n \geq 1.$$

Using this computation, it was shown in my work with Hom, Park, and Stoffregen [HKPS23] (published in *Geometry & Topology* in 2023) that there exist infinitely many rationally slice knots which form a linearly independent subset in the smooth concordance group $\mathscr{C}$. As a result, we were able to show that $\ker \psi$ is very far from being torsion.

Theorem 4.1 ([HKPS23]). The kernel of $\psi : \mathscr{C} \to \mathscr{C}_{\mathbb{Q}}$ contains a $\mathbb{Z}^\infty$ subgroup.

This result was then generalized in two different works. In my joint work [KP22] with Park, I was able to prove, using techniques developed in [Kan22], that this linear independence is not a special phenomenon for the figure-eight knot, but rather a very general phenomenon which occurs for "half" of knots which are torsion in $\mathscr{C}$.

Theorem 4.2 ([KP22]). There exists a nontrivial group homomorphism $\mathfrak{A} : \mathscr{C}_{tor} \to \mathbb{Z}_2$, where $\mathscr{C}_{tor}$ denotes the torsion subgroup of $\mathscr{C}$, such that for any knot K not contained in the kernel of $\mathfrak{A}$, the set of cabled knots $\{K_{2n+1,-1} \,|\, n \geq 1\}$ contains a linearly independent infinite subset in $\mathscr{C}$.

Moreover, in my work [HKP23] with Hom and Park, we were able to show, also using the techniques from [Kan22], that we can actually find a linearly independent (in $\mathscr{C}$) infinite set of knots which are both rationally slice and topologically slice.

Theorem 4.3 ([HKP23]). The intersection $\ker \phi \cap \ker \psi$ contains a $\mathbb{Z}^\infty$ subgroup. In fact, the set of topologically and rationally slice knots

$$\{(\mathrm{Wh}^+(3_1)\sharp 4_1)_{2n+1,1}\sharp - (\mathrm{Wh}^+(3_1))_{2n+1,1}\sharp - (4_1)_{2n+1,-1} \,|\, n \geq 1\}$$

admits a linearly independent infinite subset in $\mathscr{C}$.

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Towards a categorical interpretation of holomorphicity

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Topological dynamical systems

A *dynamical system* is a pair (X, f) of a set X (usually equipped with a structure such as topology or algebra) and a self-map $f : X \to X$ which describes the behavior of points of the set over (discrete) time. Examples include the motion of particles in a room and the time evolution of points in the phase space according to the Hamilton's equations. The power of the theory of dynamical systems is due to its ubiquity—whenever you have a set and a self-map, you can use the theory of dynamical systems, for instance, to extract some interesting quantity.

One of the major questions in the study of dynamical systems is how to measure the complexity of a dynamical system (X, f). Arguably the most important such measurement is the *topological entropy* $h_{\mathrm{top}}(f)$ introduced by Adler, Konheim and McAndrew [AKM65] in 1965. It is a nonnegative real number which measures the complexity of a *topological dynamical system*, i.e., a pair of a topological space and a continuous self-map. Most of the interesting topological dynamical systems have positive topological entropy. For example, the Dehn twist along a simple closed curve on a surface has zero topological entropy while the composition of two Dehn twists along two different curves can have positive topological entropy depending on their intersection pattern.

To study a dynamical system is a hard task since, most of the time, it is highly nonlinear. Therefore we try to *linearize* a dynamical system and extract some information of the dynamical system from the linearization. In the case of a topological dynamical system (X, f), we consider a linearized dynamical system $(H^*(X), f^*)$ obtained by taking cohomology. A celebrated theorem by Gromov [Gro87] and Yomdin [Yom87] says that the topological entropy of a holomorphic dynamical system can be computed from its linearization:

Theorem 1 (Gromov Yomdin theorem [Gro87, Yom87]). For a compact Kähler manifold X and a holomorphic diffeomorphism $f : X \to X$, we have

$$h_{\mathrm{top}}(f) = \log \rho(f^*)$$

where $\rho(f^*)$ denotes the spectral radius of f^*, i.e., the maximum of the absolute values of the eigenvalues of f^*.[*]

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[*] The ≥ part holds for a smooth compact manifold X and a diffeomorphism $f : X \to X$ [Yom87]. The *holomorphicity* is needed for the ≤ part [Gro87].

Categorical dynamical systems

I am mostly interested in topological dynamical systems with complex or symplectogeometric origin, i.e., topological dynamical systems (X, f) where X is a complex manifold or a symplectic manifold and $f : X \to X$ is a holomorphic diffeomorphism or a symplectomorphism. In both cases, we can *categorify* the dynamical system by taking the *derived category* $(D^b\mathrm{Coh}(X), \mathbb{L}f^*)$ or the (derived) *Fukaya category* $(D^\pi\mathrm{Fuk}(X), f_*)$ respectively. These categories have *triangulated structures* and thus, by categorifying such a dynamical system, we can make use of homological algebraic techniques to study the dynamical system.

More generally, we can consider a *categorical dynamical system*, i.e., a pair $(\mathscr{D}, \Phi)$ of a triangulated category $\mathscr{D}$ and an exact endofunctor $\Phi : \mathscr{D} \to \mathscr{D}$. The study of categorical dynamical systems was initiated by Dimitrov, Haiden, Katzarkov and Kontsevich [DHKK14] in 2014. In particular, they introduced the categorical entropy $h_{\mathrm{cat}}(\Phi)$ as a categorical analogue of the topological entropy. It is now known that the topological entropy and the categorical entropy coincide for many dynamical systems coming from complex or symplectic geometry. Therefore it is natural to ask whether an analogue of the Gromov–Yomdin theorem holds for categorical dynamical systems.

A categorical dynamical system $(\mathscr{D}, \Phi)$ can be linearized by taking the (numerical) Grothendieck group $(\mathcal{N}(\mathscr{D}), [\Phi])$. Thus a *categorical Gromov–Yomdin theorem*, if such a theorem exists, should assert that

$$h_{\mathrm{cat}}(\Phi) = \log \rho([\Phi])$$

holds under an assumption which can be regarded as a (partial) *categorification of holomorphicity*. The problem is how to interpret the holomorphicity in categorical language.

Homological mirror symmetry and stability conditions

One of the greatest discoveries in 20th century mathematics is the discovery of the *mirror symmetry*. It was first discovered by physicists studying string theory as a duality between two types of string theories associated to two different Calabi–Yau manifolds. The mirror symmetry drew attention from mathematicians around 1990 when Candelas, de la Ossa, Green and Parkes [CdlOGP91] showed that it can be used to prove (in the sense of physics) an open problem in enumerative geometry. Soon after, in his 1994 ICM address, Kontsevich [Kon95] proposed a mathematical formulation of the mirror symmetry which is now called the *homological mirror symmetry conjecture*.

The homological mirror symmetry (for Calabi–Yau manifolds) can be stated as follows: for a Calabi–Yau manifold X, there exists another Calabi–Yau manifold $X^\vee$ satisfying

$$D^\pi\mathrm{Fuk}(X) \simeq D^b\mathrm{Coh}(X^\vee)\,.$$

The Fukaya category $D^\pi\mathrm{Fuk}(X)$ only depends on the *symplectic structure* of X while the derived category $D^b\mathrm{Coh}(X^\vee)$ only depends on the *complex structure* of $X^\vee$. Thus the homological mirror symmetry can be regarded as a duality between the symplectic geometry of X and the complex geometry of $X^\vee$.

This categorical equivalence implies the correspondences between many invariants of the symplectic manifold X and the complex manifold $X^\vee$. Among others, it is expected that the homological mirror symmetry induces the correspondence between the *complex moduli space* of X and the (stringy) *Kähler moduli space* of $X^\vee$. Partly motivated by this, Bridgeland [Bri07] developed a way to extract a kind of moduli space from a triangulated category. He introduced the notion of a *stability condition* on a triangulated category and showed that the space of stability conditions admits the structure of a complex manifold. In particular, (some quotients of) the spaces of stability conditions on the Fukaya category $D^\pi \mathrm{Fuk}(X)$ and the derived category $D^b\mathrm{Coh}(X^\vee)$ are believed to give categorical descriptions of the complex moduli space of X and the Kähler moduli space of $X^\vee$ respectively.

A stability condition on a triangulated category $\mathscr{D}$ can be thought of as a slicing of $\mathscr{D}$. In order to obtain a stability condition, we have to specify which objects are *semistable* of phase $\phi \in \mathbb{R}$ and assign the *central charge* $Z(E) \in \mathbb{R}_{>0}e^{i\pi\phi}$ to each semistable object E of phase ϕ. The most important condition of the definition of a stability condition, called the Harder–Narasisimhan property, says that every object of $\mathscr{D}$ can be sliced into finitely many semistable objects.

In the case of the Fukaya category $D^\pi \mathrm{Fuk}(X)$ of a Calabi–Yau manifold X, Bridgeland [Bri07] and Joyce [Joy15] gave a conjectural description of a stability condition on $D^\pi \mathrm{Fuk}(X)$. It can roughly be stated as follows: each holomorphic volume form Ω on X determines a stability condition σ_Ω on $D^\pi \mathrm{Fuk}(X)$ whose semistable objects are special Lagrangian submanifolds L and central charge is given by the period integral $\int_L \Omega$.

Gromov–Yomdin type theorem for categorical entropy

Returning to the discussion on categorical dynamical systems, let us consider a categorical dynamical system $(D^\pi \mathrm{Fuk}(X), f_*)$ coming from a symplectic dynamical system (X, f) where X is a Calabi–Yau manifold. A natural expectation is that a categorical Gromov–Yomdin theorem should hold if f is *holomorphic for some complex structure* on X. Now suppose f is holomorphic for some complex structure on X which determines a holomorphic volume form Ω (up to scalar). Then the holomorphicity of f implies that

$$f^*\Omega = e^{-i\pi\psi}\Omega$$

for some $\psi \in \mathbb{R}$. In terms of the stability condition σ_Ω determined by Ω, this can be interpreted as

$$f_* \cdot \sigma_\Omega = \sigma_\Omega \cdot \psi$$

using the left action of the autoequivalence group and the right action of $\widetilde{GL}^+(2,\mathbb{R})$ ($\supset \mathbb{R}$) on the space of stability conditions.

Based on this idea, Barbacovi and I [BK23] formulated and proved a categorical analogue of the Gromov–Yomdin theorem:

Theorem 2 (categorical Gromov–Yomdin theorem [BK23]). Let $(\mathscr{D}, \Phi)$ be a categorical dynamical system. Suppose there are a stability condition σ on $\mathscr{D}$ and $g \in \widetilde{GL}^{+}(2,\mathbb{R})$ such that

$$\Phi \cdot \sigma = \sigma \cdot g\,.$$

Then, under some mild assumption, we have

$$h_{\mathrm{cat}}(\Phi) = \log \rho([\Phi])\,.$$

Since the introduction of the categorical entropy, many categorical dynamical invariants have been introduced: the mass growth [Ike21], the shifting number [FF23] and their polynomial analogues [FFO21, BK23]. Our work [BK23] also contains related results about these invariants and also about the stable translation length of the action of an exact autoequivalence on (a quotient of) the space of stability conditions.

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Greatest Ricci lower bounds of projective horospherical manifolds of Picard number one

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One main driving force in the development of complex geometry is the existence of *Kähler–Einstein metric*, the K-stability. We focused on the K-stability problem for higher dimensional varieties which are not toric ones, for non-homogeneous projective horopherical manifolds of Picard number one. Since the automorphsism group of a non-homogeneous projective horopherical manifold X is non-reductive, X does not admit any Kähler–Einstein metric due to Masushima in [Mat57, Théorème 1]. Hence, we can ask that "How much X is close to Kähler–Einstein?" and we calculate *the greatest Ricci lower bounds* of X which is a numerical measure that measures how close X to be Kähler–Einstein.

Let G be a semisimple algebraic group and let B be a Borel subgroup. If a closed subgroup P of G contains the Borel subgroup, we call P a parabolic subgroup. Since G/P is a smooth projective homogeneous complete variety and contains a rational curve, we call G/P is *a rational homogeneous manifold*. In a similar way, we can define a horospherical manifold. Let U be an unipotent radical of G and let H be a closed subgroup of G containing U. We call H *a horospherical subgroup* and G/H is *a horospherical homogeneous space*. Since there exists a unique Borel subgroup B containing U, there is a map from G/H to a rational homogeneos manifold G/P whose fiber is isomorphic to a torus P/H. Hence, we define *a horospherical manifold* X as a smooth projective complete G-variety such that a connected reductive algebraic group G acts with an open orbit isomorphic to a torus bundle over a rational homogeneous manifold G/P. We define *rank of* X as the dimension of the torus fiber P/H. The projective horospherical manifolds of Picard number one are classified by Pasquier, and it turned out that the automorphism groups of all nonhomogeneous ones are non-reductive.

Theorem 1 ([Pas09, Theorem 0.1]). Let X be a projective horospherical manifold of Picard number one. Assume that X is nonhomogeneous. Then it is of rank one and X is uniquely determined by its two closed G-orbits Y and Z, isomorphic to rational homogeneous manifolds G/P^{α} and G/P^{β} associated with simple roots α and β respectively, where (G, α, β) is one of the following:

(1) $X^1(n) := (B_n, \alpha_{n-1}, \alpha_n)$ with $n \geq 3$;
(2) $X^2 := (B_3, \alpha_1, \alpha_3)$;
(3) $X^3(n, k) := (C_n, \alpha_k, \alpha_{k-1})$ with $n \geq k \geq 2$;
(4) $X^4 := (F_4, \alpha_2, \alpha_3)$;
(5) $X^5 := (G_2, \alpha_2, \alpha_1)$.

Moreover, the automorphism group is of the form $(G \times \mathbb{C}^*)/C \ltimes V$ for some representation space V of G, here C is the center. In particular, the automorphism group is a connected non-reductive linear algebraic group.

Let X be a projective nonhomogeneous horospherical manifold of Picard number one with the open orbit G/H. Then there exist a map from G/H to $G/(P^{\alpha} \cap P^{\beta})$ and two projections from $G/(P^{\alpha} \cap P^{\beta})$ to $Y = G/P^{\alpha}$ and $Z = G/P^{\beta}$ respectively. Then G acts transitively on G/H, Y and Z and $\mathbb{C}^*$ acts on the fiber of the map G/H to $G/(P^{\alpha} \cap P^{\beta})$. The V factor of the automorphism group $(G \times \mathbb{C}^*)/C \ltimes V$ acts on Y to the direction Z whereas it acts trivially on Z. In particular, since the automorphsism group of a non-homogeneous projective horopherical manifold X of Picard number one is non-reductive, X dose not admit any Kähler-Einstein metric. See Théorème 1 of Matsushima in [Mat57] for this. In general, Fano manifolds do not necessarily admit Kähler–Einstein metrics whereas Calabi–Yau manifolds and Kähler manifolds of general type always admit Kähler–Einstein metrics (See Aubin [Aub78] and Yau [Yau78]). The non-homogeneous projective horopherical manifolds of Picard number one are non-exmaples with Fano condition.

It is natural to ask that a Fano manifold is how much close to be Kähler–Einstein. In the paper, as a numerical measure of the extent to which a Fano manifold is close to be Kähler–Einstein, we compute the greatest Ricci lower bounds $R(X)$ of projective horospherical manifolds of Picard number one. The *greatest Ricci lower bound* $R(X)$ of a Fano manifold X is defined as

$$R(X) := \sup\{0 \leq t \leq 1 : \text{ there exists a Kähler form } \omega \in c_1(X) \text{ with } Ric(\omega) \geq t\,\omega\}.$$

Note that if X admits a Kähler–Einstein metric then the greatest Ricci lower bound $R(X)$ is equal to one. For example the value is always one for rational homogeneous manifolds G/P because they are Kähler–Einstein (see [Mat72, Section 5]). The greatest Ricci lower bound $R(X)$ of a Fano manifold X is closely related with Tian's α-invariant and the δ-invariant defined by Fujita and Odaka [FO18]. If X dose not admits a Kähler–Einstein metric, then we have $R(X) = \delta(X, -K_X)$ by [CRZ19, Theorem 5.7] and [BBJ21, Corollary 7.6] (see also [Gol20, Section 3.3]), and $R(X) \geq \alpha(X) \cdot \dfrac{\dim X + 1}{\dim X}$ by [Tia87, Theorem 2.1].

The greatest Ricci lower bounds have been considered in certain subclasses of Fano manifolds. For any toric Fano manifold X, Li [Li11] found an explicit formula, the result is extended to Fano manifolds with torus actions of complexity one by Cable [Cab19] and homogeneous toric bundles by Yao [Yao17]. When X is a smooth Fano equivariant compactifications of complex Lie groups, Delcroix [Del17] obtained a formula for the greatest Ricci lower bound, where the barycenter with respect to the Duistermaat–Heckman measure. Furthermore, the recent result of Delcroix and Hultgren [DH21] extends the formula to the case of *horosymmetric* manifolds introduced in [Del20b], which is a class of spherical varieties including horospherical manifolds and smooth symmetric varieties.

Since horospherical manifold X is spherical, it has an open B-orbit, say X^0. Since B stabilizes the open B-orbit X^0, the stabilizer P at $y \in X^0$ under the G-action is a parabolic subgroup of G. Then, we have the algebraic Levi decomposition $P = L \ltimes P^u$, where P^u is the unipotent radical of P and L is the Levi factor of P. Let Φ^+ be the set of positive root of G, Φ_L be the set of roots of L and Φ_{Pu} be the set of roots of the unipotent radical P^u. Let ρ_G be the sum of all fundamental weights of the root system for G, ρ_L be the sum of all fundamental weights of the root system for the Levi factor L and let ρ_P be the half of the sum of roots of P.

Then we have

$$2\rho_P = \sum_{\alpha \in \Phi_{Pu}} \alpha = \sum_{\alpha \in \Phi^+ \setminus \Phi_L} \alpha = 2\rho_G - 2\rho_L .$$

Proposition 1 ([DH21, Corollary 1.3]). The greatest Ricci lower bound $R(X)$ of a Fano horospherical manifold X is equal to

$$\sup \left\{ t \in (0,1) : 2\rho_P + \frac{t}{1-t}(2\rho_P - \mathrm{bar}_{DH}(\Delta)) \in Relint(\Delta) \right\},$$

where Relint(Δ) means the relative interior of the moment polytope and $\mathrm{bar}_{DH}(\Delta)$ is the barycenter of the moment polytope $\Delta(X, K_X^{-1})$ with respect to the Duistermaat Heckman measure

$$\prod_{\alpha \in \Phi_{Pu}} \kappa(\alpha, p)\, dp .$$

Here, κ denotes the Killing form on the Lie algebra $\mathfrak{g}$ of G and $p \in \mathfrak{X}(T) \otimes \mathbb{R}$.

By Proposition 1, we immediately get the following elementary geometric expression for $R(X)$.

Corollary 1 ([HKP23, Corollary 2.9]). Let X be a Fano horospherical manifold without a Kähler Einstein metric. Denote by $R(X)$ the greatest Ricci lower bound of X. Let A be the point corresponding to $2\rho_P \in \mathfrak{X}(T)$ and C the barycenter $\mathrm{bar}_{DH}(\Delta)$ of the moment polytope $\Delta(X, K_X^{-1})$ with respect to the Duistermaat Heckman measure. If Q is the point at which the half-line starting from the barycenter C in the direction of A intersects the boundary of the moment polytope Δ, then we have $R(X) = \dfrac{|\overrightarrow{AQ}|}{|\overrightarrow{CQ}|}$.

Using the formula, we could calculate the greatest Ricci lower bound $R(X)$ of each nonhomogeneous projective horospherical manifold X of Picard number one as follows.

Theorem 2 ([HKP23, Theorem 1.2]). For each nonhomogeneous projective horospherical manifold X of Picard number one, its greatest Ricci lower bound $R(X)$ is as in Table 1, in which $n \geq 3$ for $X^1(n)$ and $n \geq k \geq 2$ for $X^3(n,k)$.

X	$R(X)$
$X^1(n)$	$\dfrac{n\displaystyle\int_{-n}^{2}(2-t)(n+t)^{n-1}(t+2n+2)^{\frac{n(n-1)}{2}}\,dt}{\displaystyle\int_{-n}^{2}(2-t)(n+t)^{n}(t+2n+2)^{\frac{n(n-1)}{2}}\,dt}$
X^2	$\frac{20}{21}\approx 0.952$
$X^3(n,n)$	$\dfrac{2}{(n+2)\displaystyle\int_{0}^{1}(1-t^2)^n\,dt}=\dfrac{2\times(2n+1)!}{(n+2)(2^n\times n!)^2}$
$X^3(n,k)$	$\dfrac{(2n-2k+2)\displaystyle\int_{-k}^{2n-2k+2}(k+t)^{k-1}(2n-2k+2-t)^{2n-2k+1}(4n-3k+4-t)^{k-1}\,dt}{\displaystyle\int_{-k}^{2n-2k+2}(k+t)^{k-1}(2n-2k+2-t)^{2n-2k+2}(4n-3k+4-t)^{k-1}\,dt}$
X^4	$\frac{178992099}{243545402}\approx 0.734$
X^5	$\frac{56}{67}\approx 0.8358$

TABLE 1. The greatest Ricci lower bounds.

Moreover, we could calculate the limits $\lim_{n\to\infty} R(X^1(n))$, $\lim_{n\to\infty} R(X^3(n,n))$, and $\lim_{n\to\infty} R(X^3(n,k))$.

Corollary 2 (HKP23, Corollary 1.3]). For a fixed integer $k \geq 2$,

(1) $R(X^1(n))$ converges to 1 as n increases, that is, $\lim_{n\to\infty} R(X^1(n)) = 1$.

(2) $R(X^3(n,n))$ converges to zero as n increases, that is, $\lim_{n\to\infty} R(X^3(n,n)) = 0$.

(3) $R(X^3(n,k))$ converges to 1 as n increases, that is, $\lim_{n\to\infty} R(X^3(n,k)) = 1$.

As we said above, $R(X)$ can be regarded as a measure of the extent to which a Fano manifold is close to be Kähler–Einstein and mostly known examples have values close to 1. But the greatest Ricci lower bound of *the odd symplectic Grassmannian* $X^3(n,n) = SGr(n, 2n+1)$ can be arbitrarily close to zero as n grows, thus, this gives us a very interesting sequence of examples.

I would like to thank DongSeon Hwang and Kyeong-Dong Park for their hard works and encouragement. Based on my previous work on horospherical varieties of Picard number one, I could contribute on simplifying general theories to horospherical varieties of Picard number one. Also, I worked on the calculation of the moment polytope which is one dimensional to get the Duistermaat–Heckman measure. Most work was done through emails and on-line meetings because of the Pandamic. The work was fascinating like a puzzle at least for us.

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Quantum Hall effect

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Jack polynomial and quantum Hall state

The Hall effect, first discovered by Edwin Hall in 1879, is a phenomena observed in two-dimensional electron system subject to strong external magnetic field. A potential difference (the Hall voltage) across an electrical conductor that is transverse to an electric current. The quantum Hall effect is the quantized version of the Hall effect which the observed Hall resistance takes quantized value

$$R_{\text{Hall}} = \frac{h}{e^2\nu} \tag{1}$$

h is the Planck constant and e is the elementary charge. The divisor ν can be either integer ($\nu = 1,2,3,4,\ldots$) or fractional ($\nu = \frac{k}{r} = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots$). Roughly speaking, `$\nu$` is the filling factor of the Landau level.

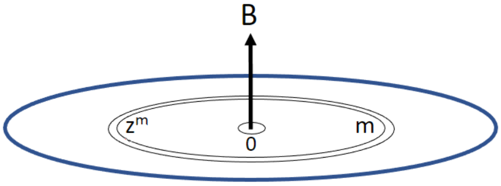

Figure 1: Orbital occupation in the Landau problem on a disk.

The simplest physical-relevant fractional quantum Hall model are anti-symmetric polynomials. It describes spin-polarized electrons in a partially filled Landau level. The same wave function can be used to study symmetric (bosonic) fractional quantum Hall states, which are obtained by multiplication with odd powers of Vandermonde determinant.

The Laughlin wavefunction, which corresponds to $\nu = 1/r$ filling factor of fractional QHE, has provied a key to understand the quantum Hall effect (QHE). It models the simplest abelian FQH and is the building blocks of model wavefuntion of more general states, both abelian and non-abelian such as Moore-Read ($\nu = \frac{2}{5}$) and Read-Rezayi state. The wavefunctions of such models, aside from the Gaussian factor which we will drop, are conformally-invariant multivariable polynomials. All three of Laughlin, Moore-Read, and Read-Rezayi state wavefunctions are proven to be special cases of the Jack polynomial $J_{\mathbf{n}}^{\frac{1}{\kappa}}$ with the Jack parameter κ taking negative rational value.

$$\kappa = -\frac{r-1}{k+1} \tag{2}$$

Supersymmetric gauge theory in 4 dimension

The relation of low-energy physics of supersymmetric gauge theory and integrable system has been an active research for decades. One of the best-known story is the Seiberg-Witten curve of the $\mathcal{N} = 2$ supersymmetric gauge theories can be identified as the spectral curve of the integrable systems. This correspondence was later extended to the quantum level by Nekrasov and Shatashivilli, with the gauge theories subjected to the Ω -deformation. This deformation introduces two parameters z$(\varepsilon_1, \varepsilon_2)$ associated to the rotation on the two orthogonal plane in $\mathbb{R}^4 = \mathbb{C}^2$. The partition function $\mathscr{Z}$ and BPS observables can be computed exactly by localization technique for a variety of gauge theories. In the limit $(\varepsilon_1, \varepsilon_2) \to (0,0)$, the classical integrable system is recovered. The Nekrasov-Shatashivilli limit (NS-limit for short) $\varepsilon_1 \to \hbar$ and $\varepsilon_2 \to 0$ results in an $\mathcal{N} = (2,2)$ supersymmetry being preserved in the fixed plane. One expects to get the quantum integrable system.

From 4-dimension to 2-dimension

One is naturally to ask the question of computing the wavefunction of the integrable system. The stationary state wave function, in the context of Bethe/gauge correspondence, are the vacua of the two-dimensional $\mathcal{N} = (2,2)$ theory. In order to get the stationary wavefunction, we compute the expectation value of a special observable in the two dimensional theory - a surface defect in the four dimensional theory. It turns out that induction of co-dimensional two surface defect provides a powerful tool in the study of Bethe/gauge correspondence. The parameter of the defect becomes the coordinates that the wavefunction depends on. The four dimensional theory with a co-dimensional two surface defect can be realized as a theory on an orbifold. The localization computations extend so as to compute the defect partition function and expectation value of BPS observables.

Our scope is on the class of qq-characters observable in the gauge theory. The main statement by Nekrasov's BPS/CFT correspondence paper proves certain vanishing conditions for the expectation values of the qq -observables, both with or without defects. These vanishing conditions, called *non-perturbative Dyson-Schwinger equations*, can be used to construct KZ-type equations satisfied by the partition function. In the NS-limit, the KZ-equations becomes a Schrödinger-type equation satisfied by the partition function.

In this research, we reconstruct the Laughlin, Moore-Read, and Read-Rezayi QHE state functions by introducing two types of co-dimensional two surface defects: the orbifold defect and canonical surface defect.

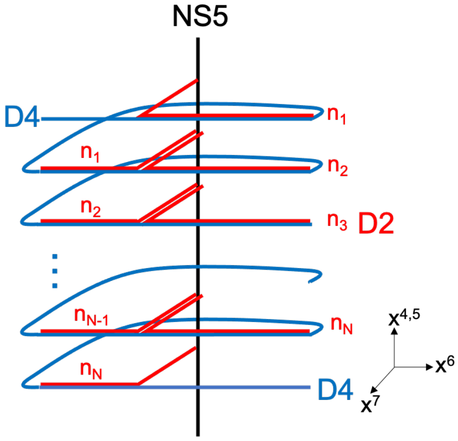

Figure 2: D-brane construction of canonical defect in 4-dimensional $\mathcal{N} = 2^*$ supersymmetric gauge theory.

It is known that the supersymmetric gauge theory instanton partition function has five and six dimensional extension. However the connection to the 2-dimensional QHE is lost in the uplift.

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Cluster algebras and Poisson Geometry

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1 A totally positive story: The discovery of cluster algebras

The story begins in the early 00's, when Sergey Fomin, Andrei Zelevinsky and Arkady Berenstein worked on the problem of total positivity in double Bruhat cells [4, 6]. A real matrix is called *totally positive* if all of its minors are positive. A set of minors is called a *test for total positivity* if whenever the minors from the test are positive on any given matrix, then all the other minors of the matrix are positive. In terms of matrices, S. Fomin, A. Zelevinsky and A. Berenstein worked on the following problem: *Describe the minimal tests for total positivity.*

Let us illustrate the problem on a square 2×2 matrix $X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$. The determinant of X together with the entries satisfy the relation

$$x_{11}x_{22} = \det X + x_{12}x_{21} \,. \tag{1.1}$$

From here, we see that if $x_{11}, x_{12}, x_{21}, \det X$ are positive, then x_{22} must be positive as well. This implies that the set of minors $\{x_{11}, x_{12}, x_{21}, \det X\}$ is a test for total positivity for 2×2 matrices. Likewise, another test is given by $\{x_{22}, x_{12}, x_{21}, \det X\}$, and the two sets can be organized in an *exchange graph* $\mathbb{T}_2$:

$$\{x_{11}, x_{12}, x_{21}, \det X\} \;\text{———}\; \{x_{22}, x_{12}, x_{21}, \det X\} \tag{1.2}$$

where the edge indicates that one can substitute the minor x_{22} for the minor x_{11} to obtain a new test.

For any other dimension, S. Fomin, A. Zelevinsky and A. Berenstein found a combinatorial description of all such tests. By means of *wiring diagrams*, their theory provided a precise way of obtaining new tests from the known ones, each time replacing one minor with another one, as above. The problem seemed to be solved. However, if one organizes the tests into an exchange graph $\mathbb{T}_3$ for 3×3 matrices, the resulting graph is not regular; that is, the vertices of $\mathbb{T}_3$ have different degrees. For instance, if we set

$$X := \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \tag{1.3}$$

then there is a test given by the following 9 minors*:

$$\{x_{13}, x_{23}, x_{33}, x_{32}, x_{31}, \det X^{[2,3]}_{[2,3]}, \det X, \det X^{[2,3]}_{[1,2]}, \det X^{[1,2]}_{[2,3]}\} \,. \tag{1.4}$$

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* Here, by X^J_I we mean a submatrix of X with the rows given by I and columns given by J.

The degree of the corresponding vertex in $\mathbb{T}_3$ equals 3: one can replace the minor x_{23} with the minor $\det X^{[2,3]}_{\{1,3\}}$, or the minor x_{32} with the minor $\det X^{\{1,3\}}_{[2,3]}$, or x_{33} with x_{22}; for each of the three obtained tests, there are other replacements that lead to new tests. So, let us replace x_{23} in (1.4) with $\det X^{[2,3]}_{\{1,3\}}$ to obtain the test

$$\{x_{13}, \det X^{[2,3]}_{\{1,3\}}, x_{33}, x_{32}, x_{31}, \det X^{[2,3]}_{[2,3]}, \det X, \det X^{[2,3]}_{[1,2]}, \det X^{[1,2]}_{[2,3]}\}\,. \tag{1.5}$$

Surprisingly, the degree of the corresponding vertex in $\mathbb{T}_3$ equals 4. One can replace $\det X^{[2,3]}_{\{1,3\}}$ back with x_{23}, or x_{33} with x_{12}, or x_{32} with $\det X^{\{1,3\}}_{[2,3]}$, or (which is new!) $\det X^{[2,3]}_{[2,3]}$ with $\det X^{[1,2]}_{\{1,3\}}$. However, there is no way of obtaining a test from (1.4) by replacing $\det X^{[2,3]}_{[2,3]}$ with some other minor. Such a minor does not exist.

Why is there such a discrepancy? The theory was done, but something was missing, the beauty of the theory was incomplete, there was no good explanation why $\det X^{[2,3]}_{[2,3]}$ is not replaceable in (1.4) but is replaceable in (1.5).

Pondering over this issue led S. Fomin and A. Zelevinsky to the discovery of a new mathematical field – the field of cluster algebras [5]. The new theory provided a universal combinatorial framework for many mathematical problems and, in particular, a way of 'completing' the exchange graphs from the problem of total positivity to regular graphs. It turned out that $\det X^{[2,3]}_{[2,3]}$ in (1.4) could be replaced with something that yields a test; however, the replacement is not a minor but some regular irreducible function in the entries of X. It is given by

$$\varphi(X) := x_{11}x_{23}x_{32} - x_{12}x_{23}x_{31} - x_{13}x_{21}x_{32} + x_{13}x_{22}x_{31}\,. \tag{1.6}$$

How does one find it? The combinatorial framework of cluster algebras tells us that (1.4) is an *extended cluster*. The elements in (1.4) that are replaceable are called *cluster variables*, and the ones that are never replaceable are called *frozen variables*[*]. The replacements in the cluster theory are called *mutations*, and they are governed by additional combinatorial data, which consists of a graph that is called a *quiver*. Each extended cluster is supplied with its own quiver, and each mutation of an extended cluster is followed by a *quiver mutation*.

Continuing with our example, let us illustrate the mutation of $\det X^{[2,3]}_{[2,3]}$ that produces φ. Let us set

$$f_{ij}(X) := \begin{cases} \det X^{[j,n]}_{[i,n-j+i]} & 1 \le i \le j \le n; \\ \det X^{[j,n-i+j]}_{[i,n]} & 1 \le j \le i \le n\,. \end{cases} \tag{1.7}$$

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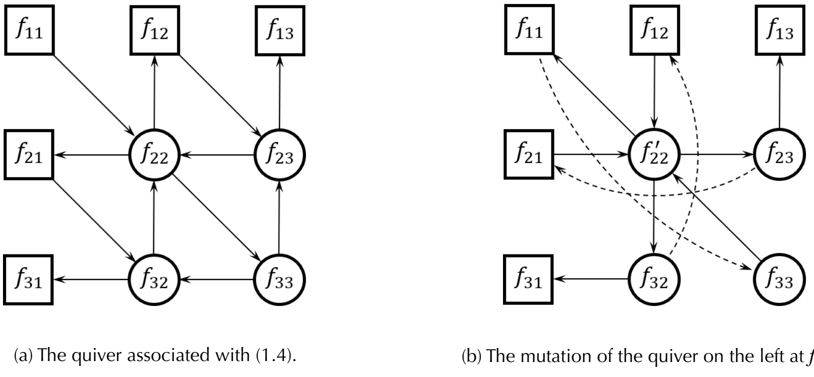
[*] One can observe from our first example with 2 × 2 matrices that, clearly, one can choose all the entries of a 2 × 2 matrix X positive in such a way that $\det X$ is negative. So, it has to be included in all the tests.



(a) The quiver associated with (1.4).

(b) The mutation of the quiver on the left at f_{22}.

Figure 1: Two quivers for a cluster structure on GL_3. Some of the arrows are dashed only for convenience. The square vertices correspond to the frozen variables.

The functions $\{f_{ij}(X)\}_{1\leq i,j\leq n}$ encompass the whole test (1.4). The quiver associated to the collection $\{f_{ij}(X)\}_{1\leq i,j\leq n}$ is illustrated in Figure 1a. The mutation of $f_{22}(X)$ is a new regular function $f'_{22}(X)$ given by the relation

$$f_{22}f'_{22} = \prod_{f_{ks}\to f_{ij}} f_{ks} + \prod_{f_{ks}\leftarrow f_{ij}} f_{ks} \tag{1.8}$$

where $f_{ks} \to f_{ij}$ means that the corresponding vertices are connected with an edge pointing towards f_{ij}, and likewise for $f_{ks} \leftarrow f_{ij}$. Plugging in the expressions for $f_{ks} \leftarrow f_{ij}$, one obtains $f'_{22}(X) = \varphi$. The quiver updates according to the following algorithm: 1) for every pair of edges $f_{ks} \to f_{22} \to f_{ij}$, draw an edge between f_{ks} and f_{ij}; 2) reverse all the edges coming in and out of f_{22}; 3) remove all 2-cycles and edges between frozen variables. Clearly, since all the functions in the right-hand side of (1.8) belong to a test on total positivity, $f_{22}(X)$ is totally positive if and only if $f'_{22}(X)$ is totally positive.

In a similar way, one mutates all the other variables. In this particular example, there is a finite number of possible extended clusters, and there are only two variables (in different clusters) that are not minors of X but that belong to some tests on total positivity.

2 The foundations and the new quests

In its abstract setting, the definition of a cluster algebra is as follows. One fixes positive integers N and M and starts with a field $\mathscr{F}$ of rational functions in $N+M$ variables with coefficients in $\mathbb{Q}$ (the *ambient field*-to-be of a cluster structure). An *extended seed* is a pair $(\mathbf{x}, B)$ that consists of a collection $\mathbf{x} := (x_1, \dots, x_N, x_{N+1}, \dots, x_{N+M})$ of elements of $\mathscr{F}$ such that $\mathscr{F} = \mathbb{Q}(x_1, \dots, x_{N+M})$, along with an integer-valued matrix $B := (b_{ij} \mid i \in [1,N],\ j \in [1,N+M])$ such that $B^{[1,N]}$ (the first N columns) is a skew-symmetric matrix*. The variables $x_1, \dots, x_N$ are called *cluster variables*, $x_{N+1}, \dots, x_{N+M}$ are called *frozen variables*, the collection $\mathbf{x}$ is called an *extended cluster* and B is called an *exchange matrix*. Given an extended seed $(\mathbf{x}, B)$ and a number $k \in [1,N]$, one can produce a new extended seed $(\mathbf{x}'_k, B')$ as follows:

Set $\mathbf{x}'_k := (\mathbf{x} \setminus \{x_k\}) \cup \{x'_k\}$ where x'_k satisfies the relation

$$x_k x'_k = \prod_{b_{kj}>0} x^{b_{kj}} + \prod_{b_{kj}<0} x^{-b_{kj}}. \tag{2.1}$$

The entries of $B' := (b'_{ij} \mid i \in [1,N], j \in [1,N+M])$ are given by

$$b'_{ij} = \begin{cases} -b_{ij} & \text{if } i = k \text{ or } j = k \\ b_{ij} + \frac{1}{2}(|b_{ik}|b_{kj} + b_{ik}|b_{kj}|) & \text{otherwise}. \end{cases} \tag{2.2}$$

Fixing an extended seed $\Sigma_0 := (\mathbf{x}_0, B_0)$ (the *initial extended seed*), one sets $\mathscr{C} := \mathscr{C}(\Sigma_0)$ to be the collection of all possible extended seeds that can be obtained from Σ_0 via iterative applications of mutations. The collection $\mathscr{C}$ is called a *cluster structure*. The *cluster algebra* $\mathscr{A}(\mathscr{C})$ is a subalgebra of $\mathscr{F}$ generated by all cluster and frozen variables from $\mathscr{C}$ with coefficients in $\mathbb{Z}$. The *upper cluster algebra* $\bar{\mathscr{A}}(\mathscr{C})$ is a subalgebra of $\mathscr{F}$ given by the intersection of the subalgebras $\mathscr{L}(\mathbf{x}) := \mathbb{Z}[x_1^{\pm 1}, \dots, x_N^{\pm 1}, x_{N+1}, \dots, x_{N+M}]$ for all possible extended clusters $\mathbf{x}$ from $\mathscr{C}$. One of the most remarkable results in the theory of cluster algebras is

Theorem 2.1. The inclusion $\mathscr{A}(\mathscr{C}) \subseteq \bar{\mathscr{A}}(\mathscr{C})$ holds.

In other words, a cluster variable from any extended cluster can be represented as a Laurent polynomial in terms of any other extended cluster.

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[*] The matrix $\begin{bmatrix} B^{[1,N]} & B^{[N+1,N+M]} \\ -\left(B^{[N+1,N+M]}_{[1,N]}\right)^t & 0 \end{bmatrix}$ is then the adjacency matrix of a quiver.

Continuing our story, once S. Fomin and A. Zelevinsky laid the foundations of the cluster theory in [5], they launched a research program dedicated to uncovering cluster structures in the coordinate rings of interesting varieties (mostly in Lie theory). This means that given a variety V over a field $\mathbb{K} \supseteq \mathbb{Q}$, its coordinate ring $\mathcal{O}(V)$ and the ring of fractions $\mathcal{F}(V)$, one sets $\mathcal{F}(V)$ to be the ambient field of a future cluster structure, and one wants to find $\mathcal{C}$ such that* $\mathcal{A}_{\mathbb{K}}(\mathcal{C}) := \mathcal{A}(\mathcal{C}) \otimes \mathbb{K} = \mathcal{O}(V)$. While the problem might be interesting on its own right, S. Fomin and A. Zelevinsky were largely motivated by the work of G. Lusztig, who found a remarkable connection between total positivity and canonical bases of quantum groups in [12, 13].

Once a cluster structure in $\mathcal{F}(V)$ is found, there are tools for proving $\mathcal{A}_{\mathbb{K}}(\mathcal{C}) \subseteq \mathcal{O}(V)$. However, proving the equality is a much more difficult task. For this reason, A. Berenstein, S. Fomin and A. Zelevinsky introduced in [3] the notion of an upper cluster algebra, and now one strives to achieve at least $\bar{\mathcal{A}}_{\mathbb{K}}(\mathcal{C}) = \mathcal{O}(V)$. If this equality holds, we say that $\mathcal{C}$ is *complete*. For example, in the same work, they proved that the cluster structures on double Bruhat cells are complete, but it is still not known whether $\mathcal{A}_{\mathbb{K}}(\mathcal{C}) = \mathcal{O}(V)$ holds for them. The equality holds, for instance, for a cluster structure on the Grassmannian $\mathrm{Gr}_{2,n+2}(\mathbb{K})$, $n \geq 1$, in which each extended cluster is modelled via Plücker coordinates and the mutations are Plücker identities†, as well as for the cluster structure on GL_3 introduced in the previous section.

3 A surprising entrance of Poisson geometry

Around the same time the cluster algebras were discovered, Misha Gekhtman worked in the field of integrable systems and learned about the results of S. Fomin and A. Zelevinsky on total positivity in double Bruhat cells [6] and the cluster structures associated to them [3]. For no particular reason, Misha Gekhtman decided to compute Poisson brackets between the variables in the initial extended clusters from the problem of total positivity. Surprisingly, all the extended clusters $\mathbf{x}$ exhibited a very special property: for each $x_i, x_j \in \mathbf{x}$, $\{x_i, x_j\} = \omega_{ij} x_i x_j$ for some constants $\omega_{ij} \in \mathbb{Q}$. When this property holds, $\mathbf{x}$ is called *log-canonical*, and if every extended cluster of a given cluster structure $\mathcal{C}$ is log-canonical, $\mathcal{C}$ is called *compatible* with the Poisson bracket. Now, why are the tests on total positivity accompanied with this peculiar Poisson property of log-canonicity? What does one field have to do with another?

A collaboration of M. Gekhtman, M. Shapiro and A. Vainshtein in their groundbreaking paper [8] found a way of constructing cluster structures $\mathcal{C}$ from log-canonical collections of regular functions $\mathbf{x} := (x_1, x_2, \ldots, x_{N+M})$. Let us set $\Omega := (\omega_{ij})_{i,j=1}^{N+M}$ where ω_{ij} are given from $\{x_i, x_j\} = \omega_{ij} x_i x_j$. They found that if one can solve the equation $B \cdot \Omega = [I \quad 0]$ with respect to a matrix B of size $[1,N] \times [1,N+M]$ such that $B^{[1,N]}$ is skew-symmetric, then $(\mathbf{x}, B)$ is an extended seed of some cluster structure $\mathcal{C}$. Moreover, any other extended cluster in $\mathcal{C}$ is log-canonical.

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* The equality might require an additional localization at the frozen variables if they are invertible as elements of $\mathcal{O}(V)$ (in other words, the variables are represented by nowhere vanishing regular functions).

† One also uses Plücker coordinates to show that the Grassmannian is a projective variety.

The cluster structures on Grassmannians [17] and double Bruhat cells [3] that were known by that time were found to be compatible with some Poisson brackets [8, 11]. In addition, in the work [9], M. Gekhtman, M. Shapiro and A. Vainshtein constructed cluster structures on connected simply connected simple complex algebraic groups G. They conjectured that a given group G carries multiple different cluster structures, each of which is complete and compatible with a *factorizable quasi-triangular* Poisson bracket. We refer to this conjecture as the *GSV* conjecture.

In the theory of Poisson-Lie groups, the class of factorizable quasi-triangular Poisson brackets is the only class of Poisson brackets (compatible with the group structure) that has been classified. In a certain sense, the classification becomes wild beyond this class (it is akin to the classification of simple complex Lie algebras versus the nilpotent Lie algebras). As was shown by Belavin and Drinfeld in [1, 2], for a given group G, a set of simple roots Π and the Cartan subalgebra $\mathfrak{h}$, the moduli space of such Poisson brackets is described by the data $(\Gamma_1, \Gamma_2, \gamma, r_0, c)$ where:

- $\Gamma_1, \Gamma_2 \subseteq \Pi$, $\gamma : \Gamma_1 \to \Gamma_2$ a nilpotent* isometry;
- A continuous parameter $r_0 \in \mathfrak{h} \otimes \mathfrak{h}$ and a scalar $c \in \mathbb{C}^*$.

The triple $\Gamma := (\Gamma_1, \Gamma_2, \gamma)$ is called a *Belavin-Drinfeld triple* (*BD triple*). The resulting compatible cluster structures do not depend on r_0 and c, so let us denote by $\{\cdot,\cdot\}_\Gamma$ a Poisson bracket associated with Γ. If $\Gamma_1 = \Gamma_2 = \varnothing$, the BD triple $\Gamma_{\mathrm{std}} := \Gamma$ is called *trivial*. The cluster structure on SL_3 (or GL_3) described in Section 1 is compatible with $\{\cdot,\cdot\}_{\Gamma_{\mathrm{std}}}$.

In addition to a group G endowed with $\{\cdot,\cdot\}_\Gamma$, there are two other objects that naturally accompany it: the *Poisson dual* of G and the *Drinfeld double* of G. The first object arises from the following observation: if $\xi, \eta \in \mathfrak{g}^*$ are elements of the dual of the Lie algebra $\mathfrak{g}$ of G and $f, g \in C^\infty(G)$ are smooth functions such that $d_e f = \xi$ and $d_e g = \eta$, then the formula

$$[\xi, \eta]_\Gamma^* := d_e\{f, g\}_\Gamma \tag{3.1}$$

defines a Lie algebra structure on $\mathfrak{g}^*$. Integrating the latter, one obtains the *Poisson dual* G^* of G. Its Lie algebra is given by $\mathfrak{g}^*$. Moreover, G^* is equipped with its own Poisson bracket $\{\cdot,\cdot\}_\Gamma^*$ which (as in (3.1)) recovers the Lie algebra structure on $\mathfrak{g}$.

The second object, the *Drinfeld double* $D(G)$ of G, comes from combining G and G^* into a single object. As a group, $D(G) := G \times G$; it is equipped with a Poisson bracket $\{\cdot,\cdot\}_\Gamma^D$. The group G is embedded into $D(G)$ as the diagonal subgroup and G^* is in some other way immersed into $D(G)$. The restrictions of $\{\cdot,\cdot\}_\Gamma^D$ to G and G^* coincide with $\{\cdot,\cdot\}_\Gamma^D$ and $\{\cdot,\cdot\}_\Gamma^*$.

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* That is, for each $\alpha \in \Gamma_1$, there exists $n > 0$ such that $\gamma^n(\alpha) \notin \Gamma_1$.

4 Compatible cluster structures on groups

The GSV conjecture that was described in the previous section has been proved for a certain subclass of BD triples on SL_n (the so-called *aperiodic* BD triples; the latest reference is [7]); for other Lie types, it was proved only for the trivial BD triples. There are some considerable changes in the Poisson geometry of the group beyond the class of aperiodic BD triples, and going into that class also requires considering a more general notion of a cluster structure (the *generalized cluster structure*) that allows more than two monomials in the mutation relation (2.1).

M. Gekhtman, M. Shapiro and A. Vainshtein in [10] also constructed cluster structures for $D(\mathrm{SL}_n)$ and SL_n^* compatible with $\{\,\cdot\,,\cdot\,\}^{D}_{\Gamma_{\text{std}}}$ and $\{\,\cdot\,,\cdot\,\}^{*}_{\Gamma_{\text{std}}}$, respectively, so the GSV conjecture naturally extends to Poisson duals and Drinfeld doubles. However, this also required considering a more general notion of a cluster structure, even though they considered the trivial BD triple.

In my published work [15], I produced generalized cluster structures on $D(\mathrm{SL}_n)$ compatible with *aperiodic oriented* BD triples. In another unpublished work [14] (in collaboration with M. Gekhtman), I constructed cluster structures on SL_n^* compatible with $\{\,\cdot\,,\cdot\,\}^{*}_{\Gamma}$ for *any* BD triple Γ, as well as produced techniques for covering other Lie types. In another unpublished work [16], I also produced new techniques that significantly reduce labor in proving that a given cluster structure is complete.

To conclude this article, let us introduce two examples of generalized cluster structures on $D(\mathrm{SL}_3)$ compatible with $\{\,\cdot\,,\cdot\,\}^{D}_{\Gamma_{\text{std}}}$ and with $\{\,\cdot\,,\cdot\,\}^{D}_{\Gamma}$ where Γ is given by $\Gamma = (\{1\},\{2\}, 1\mapsto 1)$. There are 5 types of functions, which are labelled as φ, c, f, g and h. The first three types of functions do not depend on the choice of Γ, whereas the g- and h-functions are constructed based on Γ.

What really distinguishes these types of variables are their invariance properties. For instance, let N_- and N_-' be unipotent lower triangular matrices, N_+ be a unipotent upper triangular matrix, and let $A, A' \in \mathrm{SL}_n$. Then the first three types of variables (functions) satisfy the following invariance properties:

$$\varphi(AXN_-, AYN_-) = \varphi(X,Y), \quad f(N_+XN_-, N_+YN_-'), \quad c(AXA', AYA') = c(X,Y) \tag{4.1}$$

where $(X,Y) \in \mathrm{SL}_3 \times \mathrm{SL}_3$. Moreover, the c-functions are isolated frozen variables and the Casimirs of $\{\,\cdot\,,\cdot\,\}^{D}_{\Gamma}$ for any Γ (that is, the Poisson bracket $\{c_i, p\}^{D}_{\Gamma} = 0$ for any $p \in C^{\infty}(\mathrm{SL}_3 \times \mathrm{SL}_3)$). The mutation pattern for φ_{11} is *generalized*; that is, for $D(\mathrm{SL}_3)$, the mutation of φ_{11} is given by

$$\varphi_{11}\varphi_{11}' = \varphi_{12}^3 + c_1(X,Y)\varphi_{21}\varphi_{12}^2 + c_2(X,Y)\varphi_{21}^2\varphi_{12} + \varphi_{21}^3\,. \tag{4.2}$$

As for the g- and h-functions, for the (generalized) cluster structure compatible with $\{\,\cdot\,,\cdot\,\}^{D}_{\Gamma_{\text{std}}}$, they are constructed as flag minors:

$$h_{ij}(Y) = \det Y^{[j,n]}_{[i,n-j+i]}, \quad g_{ji}(X) = \det X^{[i,n-j+i]}_{[j,n]}, \; 1 \leq i \leq j \leq n, \; (X,Y) \in \mathrm{SL}_3 \times \mathrm{SL}_3. \tag{4.3}$$

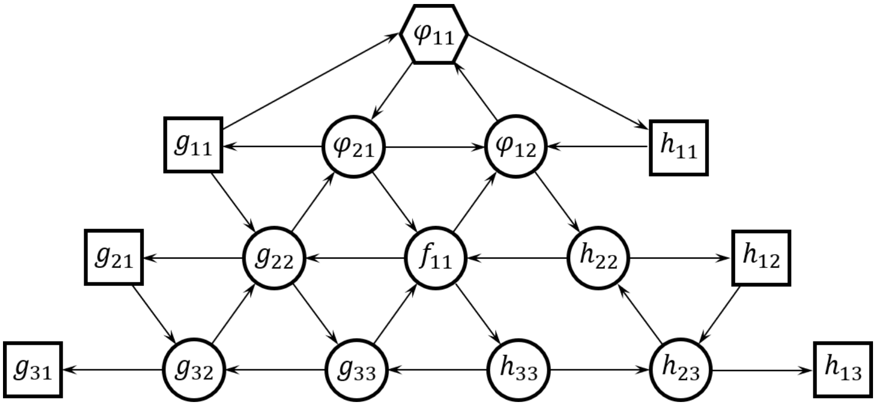

Figure 2: The initial quiver for a (generalized) cluster structure on $D(\mathrm{GL}_3)$ compatible with $\{\cdot,\cdot\}^D_{\Gamma_{\mathrm{std}}}$.

The corresponding initial quiver is illustrated in Figure 2.

In the case of $\Gamma = (\{1\}, \{2\}, 1 \mapsto 2)$, the g- and h-functions stay the same except the following ones:

$$h_{12}(X,Y) = \begin{bmatrix} y_{12} & y_{13} & 0 & 0 \\ y_{22} & y_{23} & x_{11} & x_{12} \\ y_{32} & y_{33} & x_{21} & x_{22} \\ 0 & 0 & x_{31} & x_{32} \end{bmatrix}, \; h_{23}(X,Y) = \det \begin{bmatrix} y_{23} & x_{11} & x_{12} \\ y_{33} & x_{21} & x_{22} \\ 0 & x_{31} & x_{32} \end{bmatrix}, \; h_{31} = \det \begin{bmatrix} x_{31} & x_{32} \\ y_{12} & y_{13} \end{bmatrix}. \tag{4.4}$$

The corresponding initial quiver is illustrated in Figure 3.

It is interesting that there is something like a quasi-equivalence between the two (generalized) cluster structures. There is a birational map $\mathcal{U} : D(\mathrm{SL}_3) \dashrightarrow D(\mathrm{SL}_3)$ given by the formula

$$\mathcal{U}(X,Y) = (U_r(X,Y)\cdot X \cdot U_c(X,Y), U_r(X,Y)\cdot Y \cdot U_c(X,Y)) \tag{4.5}$$

where

$$\begin{aligned} U_r(X,Y) &:= I + \frac{\det X^{[1,2]}_{\{1,3\}}}{\det X^{[1,2]}_{[2,3]}} e_{23}; \\ U_c(X,Y) &:= I + \frac{y_{12}}{y_{13}} e_{21}. \end{aligned} \tag{4.6}$$

The birational map $\mathcal{U}$ preserves the Poisson brackets (if the r_0 parts are the same), and it is also equivariant with respect to all mutations that avoid g_{21} and h_{13}. The latter variables are frozen in one cluster structure but not frozen in the other one. The map $\mathcal{U}$ is an example of what we call a *Poisson birational quasi-isomorphism*, and such maps constitute a new tool in the theory of cluster algebras in relation to Poisson geometry. I didn't use these maps to the full extent in my work [15], but I studied them in my recent preprint [16] and used them in [14]. M. Gekhtman, M. Shapiro and A. Vainshtein also used these maps in their most recent work [7].

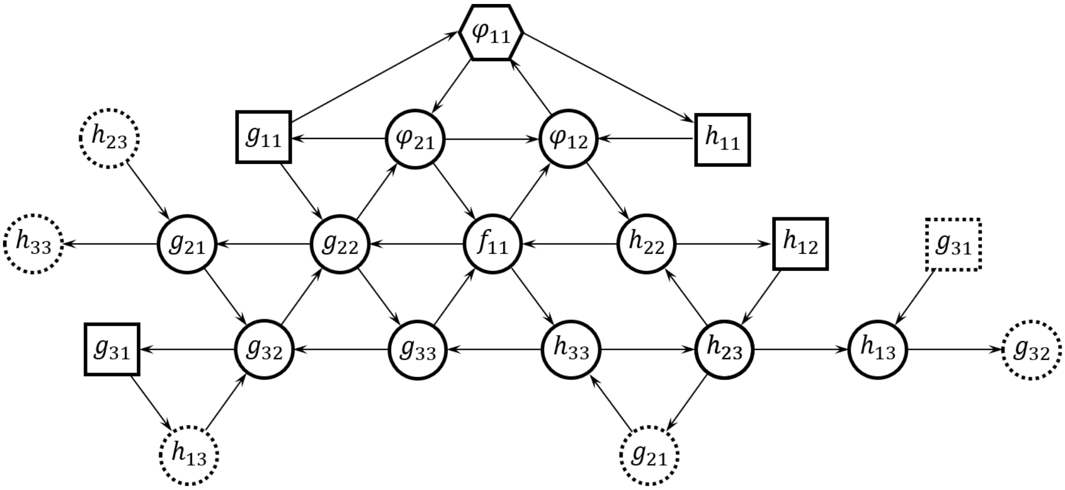

Figure 3: The initial quiver for a (generalized) cluster structure on $D(\mathrm{GL}_3)$ compatible with $\{\,\cdot\,,\cdot\,\}^D_\Gamma$ where $\Gamma = (\{1\},\{2\},1 \mapsto 2)$.

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Interviews

Morimichi Kawasaki

Associate Professor at Hokkaido University

(CGP Research member from April 2016 to March 2018)

How is your life / work after CGP?

I was a JSPS postdoctoral fellow under Professor Kaoru Ono at Research Institute for Mathematical Sciences, Kyoto University from April 2018 to March 2021. I was an assistant professor at the Department of Mathematical Sciences, College of Science and Engineering, Aoyama Gakuin University from April 2021 to August 2021. After September 2023, I am an associate professor at Department of Mathematics, Faculty of Science, Hokkaido University.

What made you decide to be a mathematician?

When I reflect on my journey towards becoming a mathematician, it's clear that my early experiences played a pivotal role. As a child, I was enrolled in the Kumon program, which significantly honed my computational skills. This not only instilled in me a confidence in tackling mathematical problems but also introduced me to the joy of studying mathematics that was years ahead of my grade level. The thrill of delving into advanced topics and understanding them was truly exhilarating. Moreover, through my proficiency in mental math gained from Kumon, my ability to visualize mathematical concepts in mind improved significantly, which later proved to be valuable in my studies and research in mathematics.

When I started university, I didn't really think about becoming a mathematician. My interests were more inclined towards the practical applications of mathematics. However, everything changed when I read John Milnor's "Morse Theory." This book showed me the fascinating correspondence between the movement of fluids on finite-dimensional manifolds and the consideration of infinite-dimensional manifolds. For instance, flows on a finite-dimensional manifold can be regarded as paths on the group of diffeomorphisms, which is an infinite dimensional manifold. It made me very interested, and I found myself drawn to the world of pure mathematics.

**What is your current math-related interest?
Please tell us about your research.**

My long-standing interest has been in the group of Hamiltonian diffeomorphisms, which are transformation groups of symplectic manifolds. Specifically, I'm fascinated by the metric structures and group-theoretic properties of this group. In terms of group-theoretic structures, I'm particularly interested in (partial) quasi-morphisms. A quasi-morphism is a real-valued function on the group satisfying certain good conditions. This concept is related to Gromov's theory of bounded cohomology and historically originates from Poincare's rotation number. Partial quasi-morphism is a generalization of quasi-morphism, which is essentially introduced by Entov and Polterovich.

I've applied various partial quasi-morphisms on the group of Hamiltonian diffeomorphisms in my research. Recently, my interest has been shifting back to metric structures, especially the Tsuboi–Kodama metric. However, even in that area, partial quasi-morphisms remain a key tool for me. Moreover, I would like to revive my research on the non-displaceable fiber of integrable systems, a topic I had set aside for several years. Partial quasi-morphisms will continue to be a key tool in this work as well. So, my research is a blend of exploring metric structures and applying partial quasi-morphisms to the group of Hamiltonian diffeomorphisms.

To conclude, I would like to share a recent research accomplishment. In a joint work with some Japanese mathematicians, I proved the following theorem:

Let S be a Riemann surface of genus at least 2. Consider f and g, symplectomorphisms on S that are isotopic to the identity, such that f and g commute, i.e., $fg = gf$. Note that the flux homomorphisms of f and g take values in $H^1(S;R)$ in this setting. Then, their cup product, which lies in $H^2(S;R)$, vanishes.

This theorem, despite its simplicity in statement, was deemed novel and was accepted this year. While the theorem itself doesn't explicitly involve Hamiltonian diffeomorphisms or quasi-morphisms, the proof relies heavily on the Py's Calabi quasi-morphism, which is defined on the group of Hamiltonian diffeomorphisms.

**What are you interested in recently?
Please share something about you.**

Now that I have a job in Hokkaido, I would like to enjoy traveling and skiing. Currently, I only have a driver's license on paper, but I want to gain actual driving experience soon so I can enjoy traveling even more. I have not been skiing for 15 years, so I would like to be able to ski again.

**We would like to hear about your dream or future plans.
Do you have a role model or a philosophy of life? :)**

I have recently started my tenure position and am currently in the process of defining new goals. However, I have some vague ideas in mind. With the newfound capacity to commit to mid-to-long-term research, I am eager to explore new avenues of study. I also aspire to contribute to the research community in various ways, such as organizing research conferences or authoring books. In the field of symplectic geometry in Japan, there are limited opportunities for young researchers to easily present their work within the community. I believe that hosting workshops and similar initiatives can help address this issue and provide a platform for emerging talents to showcase their achievements.

Eunjeong Lee

Assistant Professor at Chungbuk National University
(CGP Research member from July 2018 to February 2022)

How is your life / work after CGP?

I worked at CGP from July 2018 to February 2022 as a research fellow. The time I spent at CGP feels like it was just yesterday, but it's been nearly two years already. As of March 2022, I was appointed at Chungbuk National University, where I am now dedicated to education and research. During my time as a researcher at CGP, I didn't have to focus much on education, but now that I'm at the university, balancing teaching and guiding students has proven to be more challenging than I anticipated. I continue to struggle to find the right balance between education and research. Whenever I meet my advisor and senior professors, I am deeply inspired and constantly seek to learn how to strike a balance between research and education.

What made you decide to be a mathematician?

I have fond memories of loving mathematics from a young age, influenced by my grandfather, who was a math teacher. After enrolling at KAIST, I had many thoughts about whether to continue studying pure mathematics or pursue a more practical field. However, during the graduation project* under the guidance of Professor Dong-Youp Suh, I experienced how various mathematics I had learned in my undergraduate courses could be applied in actual research. What truly captivated me at the time (and still does) was the beautiful connections between topological spaces, combinatorics, and representation theory present in the theory of toric varieties.

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* In KAIST, it was a requirement to work on a graduation project with an advisor for one semester to complete the undergraduate program.

What is your current math-related interest? Please tell us about your research.

My research focuses on studying the properties of a manifold having group actions, specifically examining the connection between topology, geometry, combinatorics, and representation theory in the context of toric manifolds or manifolds having torus actions. My research has focused on exploring and enlarging the connections among different areas of mathematics, as provided in the case of toric manifolds. Especially, I have been interested in the topology and geometry of flag varieties, which are smooth projective variety having torus action. For instance, the combinatorics of Newton-Okounkov bodies associated with full flag varieties and the toric degenerations associated to them, and interesting subvarieties of full flag varieties admitting torus actions. A flag variety itself is not a toric variety in general, but there are interesting toric subvarieties, for instance, toric Schubert varieties and toric Richardson varieties. I have been classifying these toric varieties and their topological/geometric properties. Moreover, recently, I am interested in regular semisimple Hessenberg varieties, which are subvarieties of flag varieties having torus actions, and which provide an interesting connection between their equivariant cohomology rings and chromatic quasisymmetric functions.

What are you interested in recently? Please share something about you.

Honestly, I don't have any particular hobbies at the moment. I'm thoroughly enjoying spending my time with my 9-and-a-half-year-old first child and my 5-year-old second child, playing with LEGO, creating toys out of paper (like train models, car models, and origami), and building intricate train tracks together.

We would like to hear about your dream or future plans. Do you have a role model or a philosophy of life? :)

“The person who loves what they do, excels at it, and is currently doing it is a happy person” (my mother’s words). I believe that discovering what I love is the first step in the journey of finding happiness. Afterward, excelling at that pursuit and the ongoing struggle to persist in it is a journey of happiness in itself. When facing difficult situations and problems, I think that deeply contemplating why I truly love what I do and what makes it special can be helpful in making significant choices.

Is there anything you want to tell younger mathematicians?

If I were to say something, it would be “If you want to go far, go together”, which is known as an African proverb.

Seul Bee Lee

Research member since September 2022

How is your life in CGP / Pohang / Korea?

When I started the position, I was concerned about whether I could adapt well to the unfamiliar city of Pohang. However, with the help of kind administrative staff, I was able to settle here smoothly. Now I am enjoying the warm weather and fresh seafood of Pohang. The staff also provided generous support for research activities. I am happy to have a quiet space to focus on my research. I am kind of introverted, and I was grateful that my colleagues approached me first. My horizons expanded as I could interact with colleagues from various fields other than my research area.

What made you decide to be a mathematician?

When I was young, I grew to like mathematics due to the exhilaration of solving problems. During my undergraduate years, I liked to learn how we can prove a theorem with rigorous arguments and how we study mathematical objects by abstracting. From the start of my research during graduate school until now, I have felt attracted to finding links across various fields.

What is your current math-related interest? Please tell us about your research.

The main object of my research is the continued fraction. The continued fraction is a way to express a real number with an infinite or finite series of integers. This numerical object looks simple but it contains abundant contents. One significant aspect is that the continued fraction provides the best Diophantine approximations. Moreover, the continued fractions can be interpreted as a coding of a geodesic flow of the modular surface. We can understand the behavior of the geodesic flow by this interpretation. These concepts are generalized to various versions of continued fractions. One way of generalizing continued fractions involves considering a Fuchsian group and exploring the continued fraction associated with the group. This exploration could then explain the behavior of a geodesic flow on a hyperbolic surface, which is identified as the quotient space derived from the Fuchsian group.

What are you interested in recently?
Please share something about you.

I like music and films. Previously, my hobbies included listening to music, attending concerts, and watching movies and TV series. I take pleasure in collecting physical albums and supporting artists. I am attracted by engaging narratives and strong performances. However, the experience of pregnancy and childbirth changed my life completely. Throughout my pregnancy, my doctor advised against exposure to noisy environments. After giving birth, taking my own time has not been easy. Nevertheless, I have no complaints. My primary focus now is on my little girl. Raising my daughter brings me a sense of fulfillment, and her smile brings me happiness.

We would like to hear about your dream or future plans.
Do you have a role model or a philosophy of life? :)

The professors I met during my undergraduate and graduate studies, as well as during my postdoc period, have been a great influence on me. They are all passionate researchers as well as considerate educators. I have grown through their valuable advice. I still have a long way to go, but becoming an independent researcher is my goal. It is also my goal to be a good teacher for future students.

Anderson Vera

Research member since December 2022

How is your life in CGP / Pohang / Korea?

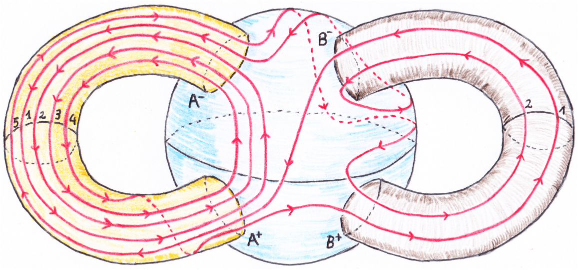
It is a great privilege to belong to the CGP because it is a place where there are many opportunities and support for carrying out academic activities. One of the things I like most about being at CGP is its location, I love the fact that I can look out my office window and see the trees moving with the wind and listen to the birds that sing in the mornings during the summer, being able to see the sky on the horizon and then sit at the desk studying mathematics. Overall, my life in Korea, particularly in Pohang, is very calm and pleasant. If it weren't for the loneliness that can sometimes be felt when being away from family, it would be like a dream place. I feel very lucky to be able to live part of my life in this place.

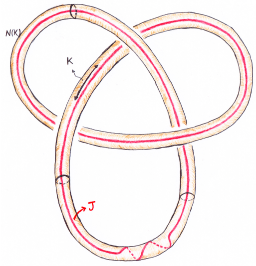
What made you decide to be a mathematician?

In my youth I did not have much idea of what it meant to study mathematics, the idea I had at school was to do calculations and many operations, although I liked doing it I did not find anything really exciting in it. After school I started studying electronic engineering at the university, there I had the opportunity to meet a special professor in the linear algebra class. I remember with joy learning for the first time an abstract idea in mathematics, the notion of vector space, and it was fascinating to be able to understand that "zero" is not necessarily the number zero. It was from that moment that the desire to want to learn mathematics arose and then I decided to change my study program and so I started studying mathematics. Now a little more than 10 years have passed since that moment and when I think about what makes mathematics attractive I think about the possibility it offers to expand imagination and creativity but not in an arbitrary way but in a more structured way, allowing to develop a critical capacity for reflection on all aspects of life.

What is your current math-related interest?
Please tell us about your research.

My current interests are knot theory and 3-manifolds. As its name suggests, knot theory involves the study of knots, such as the ones we tie in our shoes every day. And 3-manifolds are like the possible models of 3-dimensional worlds. It is fascinating to know that there are hundreds of different 3-dimensional worlds and trying to imagine and differentiate them is what I try to do in my research. Imagining these objects can be difficult, so using great creativity some mathematicians have managed to describe methods in which these objects can be built from basic pieces, such as small Lego pieces, and these small basic pieces can be imagined and even made into real models. I really enjoy trying to draw such objects and model them in real life.





What are you interested in recently? Please share something about you.

I come from a country where there are no seasons, so one of the things I enjoy most in Korea is appreciating the change of nature over time, in particular, spring and autumn are incredible. One of my favorite activities is going for a slow walk and appreciating the trees, flowers and the sky. Other things I enjoy are reading, practicing martial arts and drawing. I don't travel much, but sometimes I like to visit the Buddhist temples and the mountains around them.

We would like to hear about your dream or future plans. Do you have a role model or a philosophy of life? :)

Regarding a philosophy of life, I try to live doing as little damage as possible to nature, cultivating empathy for everything that surrounds me. I also like to make each day have something special that makes it different: deeply appreciate nature or poetry or art. Regarding future plans, it is not easy to give an answer, sometimes I dream of returning to my country of origin and having the opportunity to share there the experiences and learnings that I have had in other places. What is certain is that I want to be able to have a stable work situation and thus be able to focus on the development of various aspects of life: academically, professionally and personally. Someday I want to be able to take care of a garden with some big trees and to sit there and read or think about mathematics or life.

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