IBS Center for Geometry and Physics

CGP Walk

— Beyond the horizon —

The Second Issue

2021

기초과학연구원
Institute for Basic Science

10th ANNIVERSARY
A Decade of New Discoveries
ibs Institute for Basic Science

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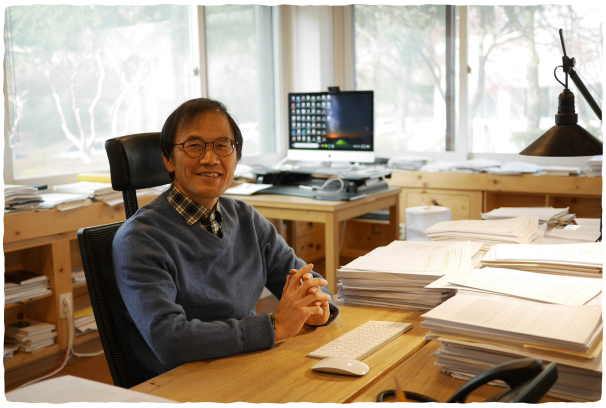
Greetings : Director's Note



COVID-19 still continues but life goes on. We met this great challenge but did not fold. I am very happy that we have been able to successfully turn this challenge into a new opportunity of upgrading CGP's scientific activities. While the pandemic largely obstructed hosting on-site international conferences and workshops, CGP's scientific activities have been upgraded by strengthening its weekly online seminar series.

As a director, it is satisfying to see that all of our three weekly online seminar series,

- Symplectic Monday,
- Tuesday Algebraic Geometry, and
- Friday Mathematical Physics,

are able to secure internationally renowned mathematicians and physicists as their speakers who otherwise might not have been able to visit CGP in person and deliver lectures. During this pandemic, our seminars became much more energized, international and open to outside scientific community.

An important mission of CGP, or other similar scientific institutes likewise, is to help the scientific community to which it belongs foster independent collective scientific intelligence to the level of the community making their own independent judgement on the value of what they are doing and on which direction they should go next and decide the community's own fate by themselves not by others. Only after that the community will be able to promote a creative environment where a truly original scientific research is performed and to encourage beginning scientists to pursue curiosity-driven research, not dictated by what other people think.

There are quite a few signs that Korean mathematics is on the verge of making great strides in a near future: its scientific community is not only generating many high quality scientific fruits but also has keen eyes in being connected to the fast changing information-based outside world. But most importantly, we are witnessing a new wave of younger generation of Korean mathematicians who can duel top scientists in the international level. As a director, I will do my best ensuring that CGP and its members help this trend last long by promoting the atmosphere of the domestic mathematical community that encourages young scientists to follow their intellectual curiosity rather than what other people think they should do. I would say "Why do you care what other people think?" as Richard Feymann did.

Background and Vision

The Center for Geometry and Physics (CGP) was founded in July 2012 as one of the first research centers of the Institute for Basic Science (IBS). The CGP originated in a government funded award, via IBS, to the research program of its director Yong-Geun Oh. This program aims to help establish and develop the emerging field of symplectic algebraic topology through a collaborative effort by experts in fields such as symplectic geometry, dynamical systems, algebraic geometry and mathematical physics. The Center is currently evolving into an international institution with a broader scope, focusing more generally on geometry and mathematical physics.

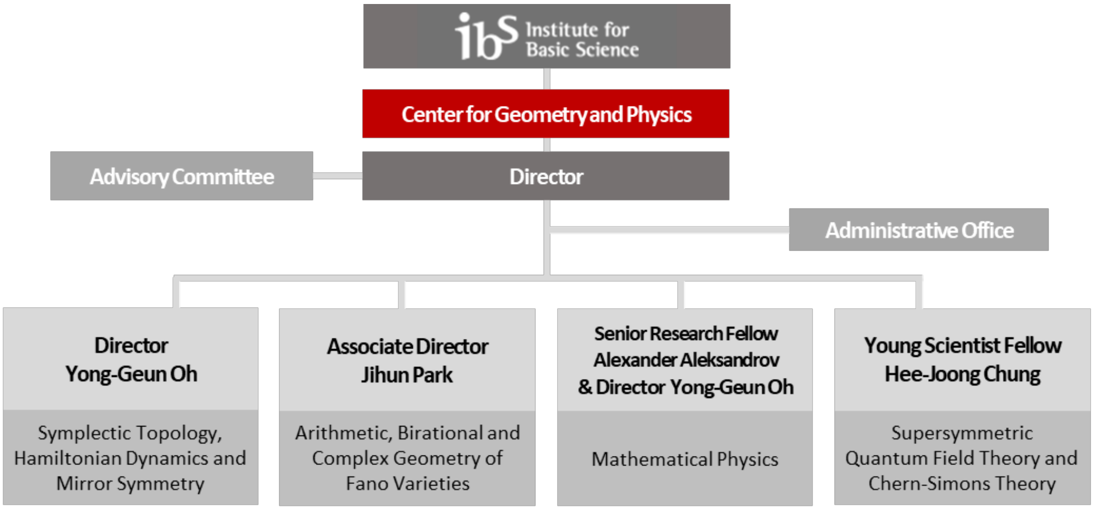
An ideal scientific research institute should be a place which fosters the disinterested pursuit of learning and the fundamental innovative thinking that advances the individual fields of inquiry of an intellectual community. The mission of the Center for Geometry and Physics (CGP) is to enable the research environment at the CGP to achieve this ideal. By now, the CGP has created such an atmosphere that ideas from mathematics and physics are naturally shared and interact. The CGP will ensure maintaining this unique research environment to serve the international community by

- Providing intellectual leadership and stewardship, guiding the development of relevant mathematics in fruitful directions by strengthening the interaction between geometry and physics,
- Playing the role of a physical nexus in Korea and beyond for the events which are the social glue of mathematical progress by hosting workshops, conferences, visitors, and so on, in order to lubricate the flow of ideas throughout the international community,
- Becoming as an incubator for young mathematicians, giving them the time and freedom to pursue ambitious and idiosyncratic research goals in a nurturant and enriching environment.

Organization

One arching research theme of the CGP is to promote interaction between symplectic geometry, algebraic geometry and mathematical physics in the study of symplectic topology and homological mirror symmetry and their applications to theoretical physics.

The CGP is organized into multi research groups, each of which comprises a senior scholar and several researchers whose areas of expertise and interest overlap synergistically.



YSF research group was terminated as of August 31, 2021.

Research Groups

Symplectic Topology, Hamiltonian Dynamics and Mirror Symmetry

Team Leader: Yong-Geun Oh

The current status of symplectic topology resembles that of classical topology in the middle of the twentieth century. Over time, a systematic algebraic language was developed to describe problems in classical topology. Similarly, a language for symplectic topology is emerging, but has yet to be fully developed. The development of this language is much more challenging both algebraically and analytically than in the case of classical topology. The relevant homological algebra of A_∞ structures is harder to implement in the geometric situation due to the analytical complications present in the study of pseudo-holomorphic curves or "instantons" in physical terms. Homological mirror symmetry concerns a certain duality between categories of symplectic manifolds and complex algebraic varieties. The symplectic side of the story involves an A_∞ category, called the Fukaya category, which is the categorified version of Lagrangian Floer homology theory. In the meantime, recent developments in the area of dynamical systems have revealed that the symplectic aspect of area preserving dynamics in two dimensions has the potential to further understanding of these systems in deep and important ways.

Research members and their research themes:

- **Elijah Fender** (The interplay of dynamics and symplectic/contact geometry)
- **Volker Genz** (Explicit problems in representation theory)
- **Hongtaek Jung** (Symplectic structures of Hitchin components and Anosov representations)
- **Sungkyung Kang** (Heegaard Floer theory, knot theory)
- **Jongmyeong Kim** (Homological mirror symmetry)
- **Seungwon Kim** (Topology and geometry)
- **Taesu Kim** (Homotopy theoretic aspects of symplectic geometry)
- **Norton Lee** (Supersymmetry, Integrable Systems, Quantum Field Theories, Mathematical Physics)
- **Sangjin Lee** (Lagrangian foliations, Symplectic mapping class group, Fukaya category)
- **Yong-Geun Oh** (Symplectic topology, Hamiltonian dynamics and mirror symmetry)
- **Yat-Hin Suen** (Complex geometry, Symplectic geometry, SYZ mirror symmetry, Homological mirror symmetry)

Arithmetic, Birational and Complex Geometry of Fano Varieties

Team Leader: Jihun Park

Fano varieties are algebraic varieties whose anticanonical classes are ample. They are classical and fundamental varieties that play many significant roles in contemporary geometry. Verified or expected geometric and algebraic properties of Fano varieties have attracted attentions from many geometers and physicists. In spite of extensive studies on Fano varieties for more than one centuries, numerous features of Fano varieties are still shrouded in a veil of mist. Contemporary geometry however requires more comprehensive understanding of Fano varieties.

Research members and their research themes:

- **Sai Somanjana Sreedhar Bhamidi** (Algebraic K-theory, algebraic cycles, algebraic stacks and derived categories)
- **Shinyoung Kim** (Complex geometry)
- **Rahul Kumar** (Analytic number theory, special functions, and the theory of partitions)
- **Eunjeng Lee** (Toric topology, Newton-Okounkov bodies, representation theory, and algebraic combinatorics)
- **Jihun Park** (Arithmetic, birational and complex geometry of Fano varieties)
- **Samarpita Ray** (Category Theory, Algebraic Geometry)
- **Sumit Roy** (Moduli of vector bundles, Hitchin system, Higgs bundles, Complex algebraic geometry, Differential geometry of bundles)
- **Haowu Wang** (Theory of modular forms and its applications)
- **Yuto Yamamoto** (Tropical geometry)

Mathematical Physics

Team Leader: Alexander Aleksandrov and Yong-Geun Oh

The mathematical relevance and deep interconnections between theoretical physics and mathematics are well-established. This subject is universally appreciated for its integrative role and for being one of the most fruitful sources of new ideas, theories and methods, and have numerous powerful applications to problems in mathematics, in particular, of geometry and topology. In recent decades, there have been various developments in supersymmetric quantum field theories and string/M-theory. In this premise, matrix models, integrable systems, Chern-Simons gauge theory, Landau-Ginzburg theory and mirror symmetry, and topological quantum field theories are the main themes of research pursued in this group.

Research members and their research themes:

- **Alexander Aleksandrov** (Mathematical physics, random matrix models, integrable systems, enumerative geometry)
- **Saswati Dhara** (Theoretical high energy physics, Chern-Simons theory in knot invariants, conformal field theory, topological field theory)
- **Yifan Li** (Algebraic geometry, algebraic topology and mathematical physics)
- **Hisayoshi Muraki** (Noncommutative geometry, nongeometric backgrounds in supergravity, discretized geometry, matrix model)
- **Abbas Mohamed Sherif** (Einstein's general relativity theory, interfacing differential geometry, geometric analysis and general relativity)

Supersymmetric Quantum Field Theory and Chern-Simons Theory

Team Leader: Hee-Joong Chung (Young Scientist Fellow)

In recent decades, there have been various developments in supersymmetric quantum field theories in various dimensions. Among many interesting advances, the correspondence between d-dimensional non-supersymmetric theories and (6-d)-dimensional supersymmetric theories that is realized from the 6d N=(0,2) theory on the product space of a d-dimensional space and a 6-d dimensional space has attracted attention both from physicists and from mathematicians. Such correspondences have been proposed for d=2, 3, 4, and they are also related to each other when 3-manifolds are boundaries of 4-manifolds or have Riemann surfaces as boundaries. When d=3, the correspondence involves the Chern-Simons theory. Many interesting aspects of the correspondences have been studied, and there are still many interesting problems. We would like to explore various aspects of the correspondences with focus on the case of d=3, 4. In addition, we would also like to explore other interesting topics in supersymmetric quantum field theories and topological quantum field theories.

The Young Scientist research project was terminated as of August 31, 2021. So the research period is September 1, 2020 – August 31, 2021 (1 year).

Research members and their research themes:

- **Hee-Joong Chung** (Quantum field theory, String/M-theory, Physical mathematics)

CGP Advisory Committee

The CGP Advisory Committee consists of eight distinguished scholars from Korea and abroad. The committee meets once a year and provides advice and input on the operations of the Center.

The current members of the Advisory Committee are:

YunSung Choi
Professor, POSTECH

Mikhail Kapranov
Professor, Kavli IPMU*

Ludmil Katzarkov
Professor, University of Miami
Professor, Higher School of Economics

JongHae Keum
President, Korean Mathematical Society
Professor, KIAS

John Morgan
Founding Director, Simons Center for Geometry and Physics

Kyewon Koh Park
Research Professor, Center for Mathematical Challenges, KIAS

Dong Youp Suh
Professor, KAIST

Herman Verlinde
Professor, Princeton University

* Kavli Institute for the Physics and Mathematics of the Universe

Research Infrastructure

The Center for Geometry and Physics aims to provide a research environment in which new and original ideas are boldly proposed, tested and revised by means of scientific interactions and communication. By doing so, we hope that some of those ideas evolve into a mature form of truly new mathematics. Thus, the goal of the center is to become the birthplace of fundamentally new research areas in addition to carrying out those projects envisioned in its initial proposal.

CGP Hall & Library

The CGP Hall is the central location of the Center's academic and social activities.

The hall serves as the venue for most of the seminars, talks and teatimes hosted by the Center. Members and visitors often gather here in small groups for discussions, exchange of ideas or simply for relaxing. It also has several offices for visitors and some members.

The CGP library collection of 4,526 books that the Center has established in topics related to the research areas of the Center greatly complements the rich archive sources available to its members and visitors. The entire CGP library collection is housed at the CGP Hall, and is open to POSTECH members as well so that students and researchers at the Department of Mathematics can take advantage of the collection.

Website, Video System, and Computing Facilities

The CGP website (http://cgp.ibs.re.kr) continues to provide schedules and information on events hosted not only by the Center but also by the Department of Mathematics and other mathematics centers at POSTECH at a glance. Also available on the website are the preprints of the members of the Center and the database for the entire collection of the CGP library which can be searched by title, author, ISBN, or year of publication.

In addition, video recordings of most talks, lectures, and conferences hosted by the CGP are uploaded and made available on the website with the consent of speakers. This feature allows anyone who is interested to access and benefit from the talks regardless of their physical location.

For the convenience of visitors and job applicants and the efficiency of the application process, the Center has implemented application features on its website. Those who are interested in visiting and conducting collaborative research with the members of the Center or who are interested in a research position at the Center can apply online at the website.

The CGP Hall, where most of the talks hosted by the Center are given, is equipped with a projector system including a Mac computer dedicated for the purpose of presentations and a Windows computer for general use.

The CGP operates a Linux-based high-performance computation server that can be used to run several CAS (computer algebra systems) such as Mathematica and Maple. The Center also provides web-publishing services for conferences and seminars as well as research-related materials and personal webpages.

CGP Guesthouse

The Center aims to facilitate the active creation of new research and the dissemination of recent progress at the boundary of what is known. CGP has created a comprehensive visitor programs to attract researchers from both Korea and abroad.

The Center operates a fully-furnished apartment-style guesthouse to better accommodate our visitors, especially those who are visiting with their family, and/or visiting for an extended period of time. The guesthouse is located near POSTECH campus, approximately 20 minutes walking away from the Center.

New IBS Research Complex in POSTECH

The Center will be relocated to the new IBS research complex building in POSTECH campus, which is scheduled to be completed in 2022. We believe that the new research complex will provide the optimal space for close research activities among researchers by integrating the research spaces. This is what CGP have wished for so long while being dispersed in three buildings of Mathematical Science building, PIAI (POSTECH Institute of Artificial Intelligence) building and RIST (Research Institute of Industrial Science and Technology) building at POSTECH campus.

The new research complex will be a five-story (1 basement level, 4 ground levels) building where all IBS-POSTECH campus research centers will move into. CGP will be located on the 3rd floor of the new research complex, and there will be private research offices for researchers, a multi-purpose hall for domestic and international academic events, small meeting rooms, and a hub lounge where welcomes social meetings and free discussions.

The new research complex will foster an environment for 'smart collaboration' by accommodating many convenient and user-friendly facilities such as various meeting rooms near the lobby, a creative lounge for guests, a cafeteria, and spots for promoting well-fare and safety of researchers. We highly expect that this will help to establish a world-class research infrastructure and attract excellent research manpower in basic science.

Scientific Activities

Statistics

- 3 conferences
- 69 talks and seminars
- 1 lecture series

CGP at a Glance

JAN

Event
Legendrians, Cluster algebras, and Mirror symmetry
(January 4 – 15)

FEB

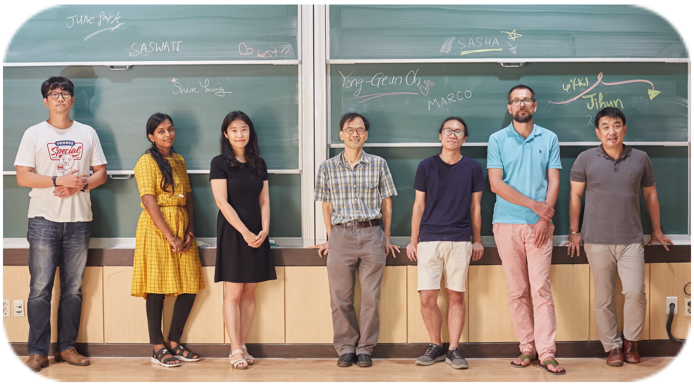
MAR

APR

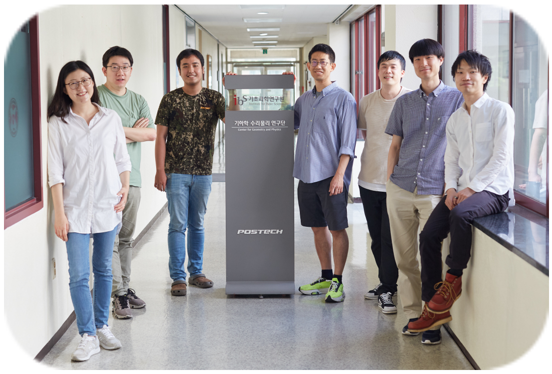
MAY

MOU
BICMR, renewed until November 5, 2026
(May 16)

JUN



2021



2021

JUL

Event
2021 Pacific Rim Complex & Symplectic Geometry Conference (July 12 – 16)

AUG

New Research Member
Sai Somanjana Sreedhar Bhamidi

SEP

New Research Member
Abbas Mohamed Sherif

OCT

New Research Member
Haowu Wang
Volker Genz
Samarpita Ray

NOV

MOU
MATRIX, renewed until November 30, 2026 (November 19)

New Research Member
Elijah Fender
Rahul Kumar
Hisayoshi Muraki

DEC

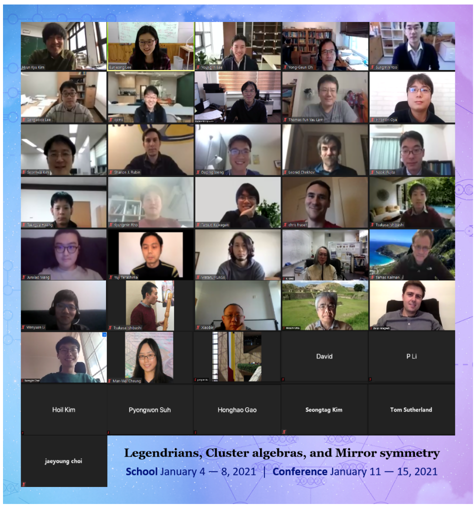
New Research Member
Sumit Roy
Norton Lee

Event
2021 Pohang Mathematics Workshop (December 2 – 4)

Conferences

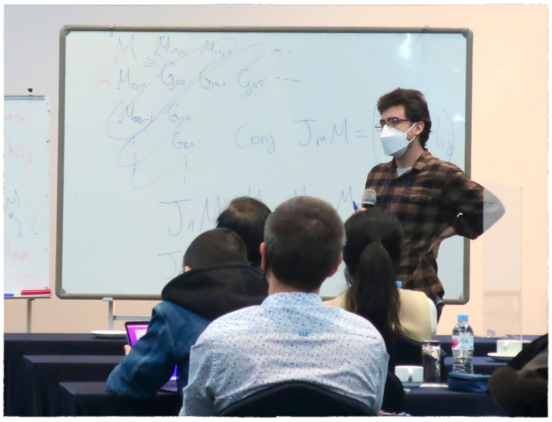
In an effort to take on a leading role in enriching the mathematical society, the Center works in collaboration with other institutes and organizations to hold conferences for a wider audience of mathematicians and scholars. The Center has organized or co-organized 3 conferences:

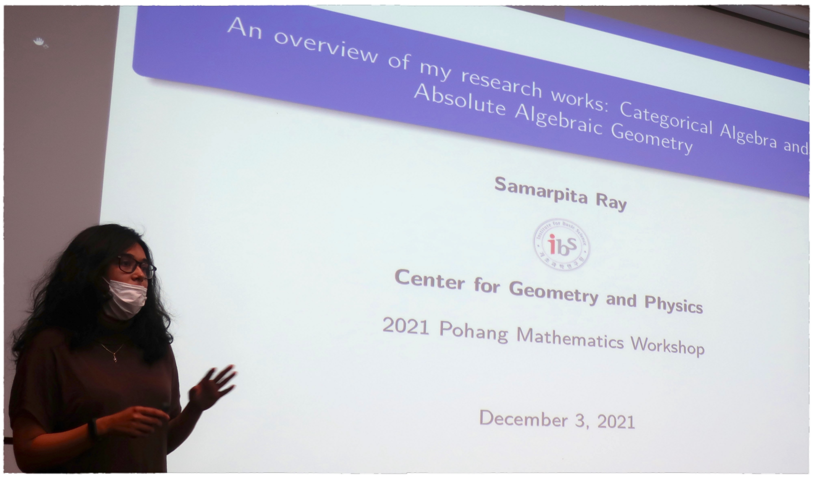
- **Legendrians, Cluster algebras, and Mirror symmetry**; January 4 – 15, 2021
  - Organizers: Byung Hee An (Kyungpook National University)
    Youngjin Bae (Incheon National University)
    Eunjeong Lee (IBS-CGP)
    Yong-Geun Oh (IBS-CGP, POSTECH)
  - Invited Speakers: Byung Hee An (Kyungpook National University)
    Roger Casals (University of California, Davis)
    Cheol-Hyun Cho (Seoul National University)
    Yunhyung Cho (Sungkyunkwan University)
    Naoki Fujita (The University of Tokyo)
    Benjamin Gammage (Havard University)
    Honghao Gao (Michigan State University)
    Hyun Kyu Kim (Ewha Womans University)
    Tatsuki Kuwagaki (Osaka University)
    Lenhard L. Ng (Duke University)
    Thomas Lam (University of Michigan)
    Sangwook Lee (Soongsil University)
    Linhui Shen (Michigan State University)
    Yat-Hin Suen (IBS Center for Geometry and Physics)
    Daping Weng (Michigan State University)
    Eric Zaslow (Northwestern University)



- **2021 Pacific Rim Complex & Symplectic Geometry Conference**; July 12 – 16, 2021
  - Local Organizers: Jun-Muk Hwang (IBS-Center for Complex Geometry)
    Yong-Geun Oh (IBS-CGP, POSTECH)
  - Scientific Committee: Xiuxiong Chen (Stony Brook University)
    Kengo Hirachi (The University of Tokyo)
    Jun-Muk Hwang (IBS-Center for Complex Geometry)
    Yong-Geun Oh (IBS CGP, POSTECH)
    Kaoru Ono (RIMS, Kyoto University)
    Yongbin Ruan (Zhejiang University)
  - Invited Speakers: Byung Hee An (Kyungpook National University)
    Youngjin Bae (Incheon National University)
    Hyungryul Baik (Korea Advanced Institute of Science and Technology)
    Huai-Liang Chang (Hong Kong University of Science and Technology)
    Kyeongsu Choi (Korea Institute for Advanced Study)
    Ryushi Goto (Osaka University)
    Genki Hosono (Tohoku University)
    Hisashi Kasuya (Osaka University)
    Ngoc Cuong Nguyen (Korea Advanced Institute of Science and Technology)
    Ryosuke Takahashi (Kyushu University)
    Peng Wu (Fudan University)
    Ziyu Zhang (ShanghaiTech University)

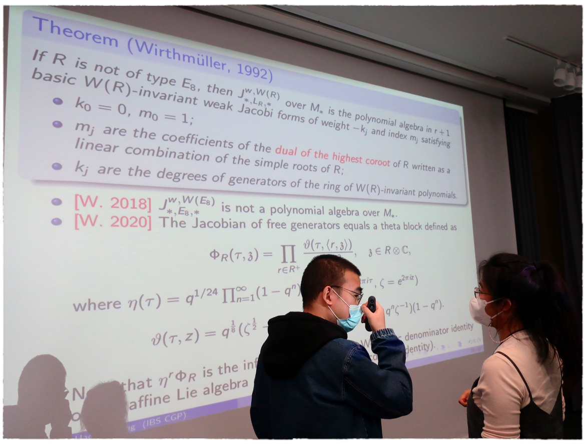
- **2021 Pohang Mathematics Workshop**; December 2 – 4, 2021
  - Organizers: Eunjeong Lee (IBS-CGP); Sangjin Lee (IBS-CGP)
  - Invited Speakers: Sai Somanjana Sreedhar Bhamidi (IBS-CGP)
    Beomjun Choi (POSTECH)
    Saswati Dhara (IBS-CGP)
    Volker Genz (IBS-CGP)
    Sungkyung Kang (IBS-CGP)
    Sangjin Lee (IBS-CGP)
    Samarpita Ray (IBS-CGP)
    Abbas Mohamed Sherif (IBS-CGP)
    Anderson Vera (POSTECH)
    Haowu Wang (IBS-CGP)






2021 Pohang Mathematics Workshop
December 2-4
2021 AVANI Central Busan Hotel, Busan
Center for Geometry and Physics


CLUSTER ALGEBRA


Theorem (Wirthmüller, 1992)

Seminars

The CGP hosts various seminars given both by visiting scholars and the members of the Center.

Derived Seminar (Mondays 13:30 – 15:30)
The derived seminar is a weekly working seminar focusing on derived tools such as model categories, ∞-categories, DG-categories, etc. The goal of this seminar is to become more familiar with these modern mathematical tools.

Symplectic Monday (Mondays 16:00 – 18:00)*
The talks are focused on symplectic geometry and chaired by Director Yong-Geun Oh.

Algebraic Geometry Seminar (Tuesdays 16:00 – 18:00)*
The talks are focused on algebraic geometry and chaired by Associate Director Jihun Park.

Wednesday Noon Seminar (Wednesdays 12:00 – 13:00)
The Wednesday Noon Seminar runs weekly with lunch for talks by CGP members on various topics of their own research interest or current works. This is kind of semi-closed seminar open to CGP members and visitors only.

Director's Seminar (Bi-weekly Wednesdays 13:30 – 15:30)
The purpose of this seminar is to give updates on current developments andmathematical research highlights in general to CGP members and visitors, and to promote deeper interaction between the speaker and the audience. The seminar's general spirit reflects that of the famous Gelfand Seminar.

The Center for Geometry and Physics Seminar (Thursdays 16:00 – 18:00)
The Center for Geometry and Physics Seminar on every Thursday afternoon is the most important regular event of the CGP, and generally all members of the Center participate. The seminars are formatted to encourage robust and dynamic interactions among participants. The seminar is structured as a two-hour talk by a designated speaker with a thirty minute intermission with tea and snack. The first half is intended to be a colloquium-level talk suitable for a general mathematical audience, while the second half can be more specialized. Discussions may continue over dinner.

Mathematical Physics Seminar (Fridays 13:00 – 15:00)*
The talks are focused on mathematical physics chaired by Director Yong-Geun Oh and a research fellow, Alexander Aleksandrov.

IBS-CGP Post-doc Lecture Series
The Director encourages CGP post-doc members to give 3-4 one-hour lectures on their research area. The main purpose of the series is to train post-doc’s lecture skills so the Director comments on the lecture series and give suggestions to improve.

* Depending on the invited speaker’s location, the online seminar was held flexibly at 10:00 – 11:00 or 16:00 – 18:00.

List of All Talks

Semi-toric degenerations of Richardson varieties from cluster algebras
Naoki Fujita (University of Tokyo)
December 13, 2021

The topology of the Gelfand–Zeitlin fiber
Jeffrey Carlson (Imperial College London)
December 6, 2021

Automorphisms of algebraic surfaces
Constantin Shramov (Steklov Mathematical Institute)
November 30, 2021

Moduli of Calabi-Yau pairs and secondary fans
Tony Yue Yu (California Institute of Technology)
November 29, 2021

CFT from Topological Recursion
Bertrand Eynard (IPHT[*]/CEA[†]-Saclay)
November 26, 2021

Smooth closing lemmas for area-preserving surface diffeomorphisms
Michael Hutchings (University of California, Berkeley)
November 22, 2021

Cohomological field theories and BKP integrability: Omega classes times Witten-classes
Danilo Lewański (Université de Genève)
November 19, 2021

Interlevel persistence and Floer theory
Michael Usher (University of Georgia)
November 15, 2021

Correlation functions for unitary invariant ensembles and Hurwitz numbers
Tamara Grava (Bristrol University, SISSA[‡])
November 12, 2021

Greatest Ricci lower bounds of smooth Fano horospherical varieties and equivariant group compactifications
Kyeong-Dong Park (Korea Institute for Advanced Study)
November 9, 2021

Intermediate Jacobians of Gushel-Mukai varieties
Alexander Kuznetsov (Steklov Mathematical Institute)
November 2, 2021

The Simplicity Conjecture
Daniel Cristofaro-Gardiner (University of Maryland)
November 1, 2021

Quantum T-Q and KZ equations in gauge theory
Nikita Nekrasov (Simons Center for Geometry and Physics)
October 29, 2021

Categorical non-properness in wrapped Floer theory
Sheel Ganatra (University of Southern California)
October 25, 2021

Bilinear expansions of lattices of KP τ-functions in BKP τ-functions, determinant and Pfaffian expressions of polynomial τ-functions
John Harnad (Mathematical Physics Laboratory, C.R.M, Université de Montréal)
October 22, 2021

Noncommutative deformations of crepant resolutions via mirror symmetry
Siu-Cheong Lau (Boston University)
October 18, 2021

Birational geometry of sextic double solids with cA points
Takuzo Okada (Saga University)
October 12, 2021

Topological recursion for generalized Hurwitz numbers
Maxim Kazarian (Higher School of Economics, Skolkovo Institute of Science and Technology)
October 8, 2021

Graph connections, (wild) character varieties and generating function in symplectic geometry
Marco Bertola (Concordia University)
October 1, 2021

Seshadri constants and K-stability of Fano manifolds
Ziquan Zhuang (MIT)
September 28, 2021

Skein valued curve counts, basic holomorphic disks, and HOMFLY homology
Tobias Ekholm (Uppsala University)
September 27, 2021

[*] Institut de Physique Théorique

[†] Commissariat à l'énergie atomique

[‡] The Scuola Internazionale Superiore di Studi Avanzati

Whittaker vectors for W-algebras from topological recursion
Aeryeong Seo (Kyungpook National University)
January 20 – 21, 2020

Virtual intersection theories
Vincent Bouchard (University of Alberta)
September 24, 2021

Kahler-Einstein smooth Fano threefolds
Ivan Cheltsov (University of Edinburgh)
September 14, 2021

The algebraic structure of groups of area-preserving homeomorphisms
Sobhan Seyfaddini (Sorbonne Université)
September 13, 2021

Mirror Symmetry Correspondence between Indecomposable Cohen-Macaulay Modules over Degenerate Cusps and Immersed Lagrangians on Surfaces
Kyungmin Rho (Seoul National University)
September 6, 2021

Tropical Lagrangian multi-sections and toric vector bundles
Yat-Hin Suen (IBS-CGP)
July 1, 2021

An algebraic model for smoothing Calabi-Yau varieties and its applications
Kwokwai Chan (The Chinese University of Hong Kong)
June 28, 2021

Tropical varieties and integral affine manifolds with singularities
Yuto Yamamoto (IBS-CGP)
June 22, 2021

Sharp Ellipsoid Embeddings and Toric Mutations
Renato Vianna (Universidade Federal do Rio de Janeiro)
June 21, 2021

KP integrability of triple Hodge integrals
Alexander Aleksandrov (IBS-CGP)
June 18, 2021

Noncompact description for Fukaya-Seidel categories of invertible curve singularities
Wonbo Jeong (Seoul National University)
June 14, 2021

Some applications of Jordan algebras in geometry
Nicolas Perrin (Versailles University)
June 8, 2021

Shifting numbers in triangulated categories
Yu-Wei Fan (University of California, Berkeley)
June 7, 2021

Minimal rational curves on Schubert varieties and their desingularizations
Michel Brion (Institut Fourier)
June 1, 2021

[Lecture Series]
The Le-Murakami-Ohtsuki invariant and Johnson-Morita theory I – VI
Anderson Vera (POSTECH-BK21 FOUR)
May 31 – June 10, 2021

Real Lagrangian tori in monotone symplectic 4-manifolds
Joontae Kim (Korea Institute for Advanced Study)
May 31, 2021

Fluxes, Holomorphic Anomalies and Elliptic Genera in d=4
Wolfgang Lerche (CERN*)
May 28, 2021

Lagrangian Cobordism and Lagrangian Surgery
Jeff Hicks (University of Cambridge)
May 24, 2021

New exact results for open enumerative invariants
Sergei Gukov (California Institute of Technology)
May 21, 2021

Lifting cobordisms and Kontsevich-type recursions for counts of real curves
Xujia Chen (Stony Brook University)
May 17, 2021

On the rationality of MUMs and 2-functions
Johannes Walcher (Heidelberg University)
May 14, 2021

Noether-Fano Inequalities and Canonical Thresholds on Fano Varieties
Charles Stibitz (Northwestern University)
May 11, 2021

Lagrangian configurations and Hamiltonian maps
Egor Shelukhin (University of Montreal)
May 10, 2021

Topologies on the exponential
Damien Lejay (IBS-CGP)
May 6, 2021

On (-1) curves in P^n and geometry of Mori Dream Spaces
Olivia Dumitrescu (University of North Carolina at Chapel Hill)
May 4, 2021

* Conseil Européenne pour la Recherche Nucléaire

The symplectic (A-infinity,2)-category and a simplicial version of the 2D Fulton-MacPherson operad
Nathaniel Bottman (Max Planck Institute for Mathematics)
May 3, 2021

Topological recursion for simple singularities
Todor Milanov (Kavli IPMU*)
April 30, 2021

Invariants of non-arborescent knots, links & topological entanglement
Saswati Dhara (IBS-CGP)
April 29, 2021

Toric varieties of Catalan type and smooth toric Richardson varieties
Eunjeong Lee (IBS-CGP)
April 27, 2021

A double Johnson filtration for the mapping class group and the Goeritz group of the sphere
Anderson Vera (POSTECH-BK21 FOUR)
April 27, 2021

Lagrangian Poincaré Recurrence via pseudoholomorphic foliations
Georgios Dimitroglou Rizell (Uppsala University)
April 26, 2021

Enumerative geometry via the moduli space of super Riemann surfaces
Paul Norbury (University of Melbourne)
April 23, 2021

Fukaya categories of quasihomogeneous polynomials
Jack Smith (St John's College Cambridge)
April 19, 2021

Abelianization, exact WKB and link invariants
Andrew Neitzke (Yale University)
April 16, 2021

Rationality of Fano 3-folds over nonclosed fields
Yuri Prokhorov (Steklov Mathematical Institute)
April 13, 2021

Scattering diagrams from blowups of toric surfaces
Hansol Hong (Yonsei University)
April 12, 2021

The Alexander polynomial as a universal invariant
Rinat Kashaev (University of Geneva)
April 9, 2021

Floer Cohomology and Arc Spaces
Mark Mclean (Stony Brook)
April 5, 2021

Loop equations and integrable hierarchies for special cubic Hodge integrals
Di Yang (University of Science and Technology of China)
April 2, 2021

Algebraic groups acting on the plane over a perfect field
Susanna Zimmermann (University of Angers)
March 30, 2021

The Rabinowitz Fukaya category and applications
Yuan Gao (University of Southern California)
March 29, 2021

Quenched free energy from spacetime D-branes
Kazumi Okuyama (Shinshu University)
March 26, 2021

Peterson conjecture via Lagrangian correspondences and wonderful compactifications
Hanwool Bae (Seoul National University)
March 22, 2021

Darboux coordinates for symplectic groupoid and cluster algebras
Leonid Chekhov (Michigan State University)
March 19, 2021

Geometric quantization and canonical metrics on polarized manifolds
Kewei Zhang (Beijing International Center for Mathematical Research)
March 16, 2021

Virtual fundamental chain in gauge theory
Kenji Fukaya (Simons Center for Geometry and Physics)
March 15, 2021

A noncommutative generalization of Witten's conjecture
Alexandr Buryak (National Research University Higher School of Economics)
March 12, 2021

Comparing numerical Iitaka dimensions
Jinhyung Park (Sogang University)
March 9, 2021

Mirror symmetry for Berglund-Hübsch Milnor fibers
Benjamin Gammage (Harvard University)
March 8, 2021

Computation of categorical entropy via spherical functors
Jongmyeong Kim (IBS-CGP)
March 4, 2021

* Kavli Institute for the Physics and Mathematics of the Universe

Visitor Programs and Visitors

CGP runs programs to support visiting scholars. The goal of the visitor programs at the center is to support dynamic researchers working in topics related to the core fields of interest. In particular, the center aims to facilitate the active creation of new research and the dissemination of recent progress at the boundary of what is known. More concretely, we believe that mixing interesting people working on interesting problems in one place has the potential to reveal commonalities, promote collaboration, and help those people advance in understanding.

The center can provide office space and housing for approved visitors. Limited funds are available to support for travel and local expenses for visiting scholars.

List of All Visitors

Yunhyung Cho (Sungkyunkwan University)
November 26 – 28, 2021

Seonjeong Park (Jeonju University)
October 22 – 28, 2021

Byung Hee An (Kyungpook National University)
September 9 – 12, 2021

Soojin Cho (Ajou University)
June 28 – July 2, 2021

MOUs

The CGP has signed several MOUs for active research collaborations and academic exchanges with the mathematics community.

Beijing International Center for Mathematical Research (BICMR), China

November 2015 – November 2026 (renewed in 2021)

- 3rd BICMR&IBS-CGP Joint Symplectic Geometry Workshop (September 24 – 26, 2019 @ POSTECH)
- Silk Road Geometry Conference 2018 (June 4 – 8, 2018 @ Gökova Geometry/Topology Institute)
- 2nd BICMR&IBS-CGP Joint Symplectic Geometry Workshop (September 18 – 22, 2017 @ BICMR)
- 1st BICMR&IBS-CGP Joint Symplectic Geometry Workshop (October 31 – November 4, 2016 @ KAL Hotel, Jeju)

Research Institute for Mathematical Sciences (RIMS), Japan

August 2017 – July 2025 (renewed in 2020)

- 2021 Pacific Rim Complex & Symplectic Geometry Conference (July 12 – 16, 2021 | Online)
- RIMS & IBS-CGP Joint Symplectic Geometry Workshop (December 2 – 4, 2019 @ Kyoto University)
- Wall-crossing Formula, Open Gromov-Witten Invariants and Related Areas (October 29 – 31, 2018 @ POSTECH)
- Pacific Rim Complex-Symplectic Geometry Conference (July 31 – August 4, 2017 @ POSTECH)

Mathematical Research Institute (MATRIX), Australia

December 2018 – November 2026 (renewed in 2021)

French-Korean International Research Laboratory (IRL) in Mathematics

January 2019 – December 2023

This platform is expected to foster French-Korean collaboration in mathematics.

French Members	Korean Members
Centre National de la Recherche Scientifique (CNRS)	IBS Center for Geometry and Physics (IBS-CGP)
École Normale Supérieure Lyon	Korea Advanced Institute of Science and Technology (KAIST)
Sorbonne Université	Korea Institute for Advanced Study (KIAS)
Université de Bordeaux	Pohang University of Science and Technology (POSTECH)
Université Grenoble Alpes	Seoul National University
Université de Paris	
Université de Strasbourg	
Institut polytechnique de Bordeaux	

Research Highlights

Alexander Aleksandrov : KdV vs. BKP

Jaehyun Kim : Flexibility for Del Pezzo Cones

Seungwon Kim : Surfaces in 4-manifolds and smooth 4-dimensional Poincaré conjecture

Shin-young Kim : Infinitesimal symmetries of weakly pseudo convex manifolds

Kyoung-Seog Lee & Kyeong-Dong Park : Ulrich bundles on Fano varieties

KdV vs. BKP

Alexander Alexandrov
Research member since February 2017

In their quest of understanding our Universe, natural sciences uncover a multitude of mathematical structures. Some are more fundamental and beautiful than others. The whole history of science tells us that such "building blocks" are inherently connected with powerful structures and symmetries, that ensure the solvability of the model, in the broad meaning of these words. One of the simplest examples of such a building block is the harmonic oscillator and the associated Heisenberg algebra. Understanding this simple system on the quantum level became a basis for the spectacular developments in physics during the last century. Thanks to their rich mathematical structure and wide range of applications, integrable systems play the role of the "harmonic oscillator of the twenty-first century."

Integrable systems are widely known to be a universal tool in theoretical physics and mathematics. They have proved to be extremely useful in such diverse areas as statistical mechanics, gauge theories in various dimensions, quantum gravity, nonlinear waves, and differential equations. They are particularly important in modern enumerative geometry including a variety of problems related to moduli spaces, Gromov–Witten invariants, and Hurwitz numbers. Here integrability plays a special role and, for some mysterious reason, it is closely related to solvability. Namely, when an enumerative geometry problem can be related to an integrable system, sooner or later it always can be completely solved.

Among different families of integrable systems, classical and quantum, integrable hierarchies of solitonic type have particularly many applications. Arguably the most important example is given by the Kadomtsev–Petviashvili (KP) hierarchy. The hierarchy is described by an infinite tower of PDEs. The first of them was introduced in 1970 by two physicists, Kadomtsev and Petviashvili, for the description of the acoustic waves in plasma. The Kadomtsev–Petviashvili hierarchy is known to be related to many other interesting integrable systems.

The Kadomtsev–Petviashvili equation was introduced as a deformation of the Korteweg–de Vries (KdV) equation, which describes waves on shallow water surfaces. More generally, the whole KdV hierarchy describes a reduction of the KP hierarchy.

The theory of integrable hierarchies was actively developed by the Kyoto school of Date, Jimbo, Kashiwara, Miwa, and Sato in the 80s of the last century. They found a fundamental relation between integrable hierarchies, representation theory of the infinite dimensional Lie algebras, and free field formalism. In particular, they described the solutions of the hierarchies in terms of tau-functions $\tau(t)$ – formal functions of infinitely many variables t_k. These functions provide a particularly convenient description in the free field formalism – they are nothing but the vacuum expectation values of some group elements. By now this theory is a classical part of mathematical physics, and new relations between its basic elements are unforeseen by the community.

In addition to the KP hierarchy, associated with $\mathfrak{gl}(\infty)$ Lie algebra, there exists its B-version (BKP), associated with the orthogonal Lie algebra $\mathfrak{go}(\infty)$. They are described by different families of free fields: charged and neutral free fermions respectively. KdV is related to affine Lie algebra $A_1^{(1)}$, which is naturally related to $\mathfrak{gl}(\infty)$ as the KdV hierarchy is a reduction of the KP hierarchy. Because of that, the KdV tau-functions are often considered in the scope of the KP formalism.

The Kontsevich–Witten tau-function of the KdV hierarchy plays a special role in modern mathematical physics. It was constructed by Witten and Kontsevich for the description of two-dimensional topological gravity. Enumerative geometry definition of this tau-function is given by the generating function of the intersection numbers on the moduli spaces of the Riemann surfaces. It is the basic building block for several universal constructions, including the Chekhov–Eynard–Orantin topological recursion and the Givental decomposition. This makes the Kontsevich–Witten tau-function one of the most well-studied tau-functions of the integrable solitonic hierarchies.

The Brézin–Gross–Witten model was introduced in lattice gauge theory 40 years ago. It has a natural enumerative geometry interpretation given by the intersection theory of Norbury's Θ-classes, also related to super Riemann surfaces. Moreover, the Brézin–Gross–Witten tau-function is another fundamental element of topological recursion/Givental decomposition, which corresponds to the hard edge case. Similar to the Kontsevich–Witten case, it is a tau-function of the KdV integrable hierarchy and can be described by the generalized Kontsevich model. All this makes the Brézin–Gross–Witten model interesting and, in many respects, similar to the Kontsevich–Witten tau-function.

There are different types of tau-functions. In particular, there is an interesting family of polynomial tau-functions. It is known that the polynomial solutions to the KP hierarchy are given by the Schur functions. These symmetric functions are the characters of irreducible representations of the general linear groups. They are closely related to the whole theory of the KP hierarchy. In particular, these functions labeled by partitions constitute a natural basis for the expansion of the solutions. The coefficients for the expansion of the Kontsevich–Witten and Brézin–Gross–Witten tau-functions are known, however, they are rather complicated and their meaning is not clear yet. For the BKP hierarchy, the analogous role of the natural basis is played by the Schur Q-functions, which were introduced by Schur for the description of the projective representations of the symmetric groups.

Recently there appear some indications that the KdV hierarchy may also have a natural relation to the BKP hierarchy. Indeed, in a recent paper by Mironov and Morozov it was noted that the Kontsevich–Witten tau-function has a simple expansion in terms of the Schur Q-functions. The coefficients of this expansion are just combinations of the Schur Q-functions at the special values of the variables. Actually, Mironov and Morozov have reformulated the older result of Di Francesco, Itzykson, and Zuber, who found the expansion of the Kontsevich–Witten tau-function in the basis of some symmetric functions. These functions were later identified with the Schur Q-functions by Józefiak.

This result was generalized by the author to a family of KdV tau-functions related to the Brézin–Gross–Witten model. Based on of these expansions, the author has conjectured (for the Kontsevich–Witten tau-function) and proved (for the Brézin–Gross–Witten tau-function) that these KdV tau-functions also solve the BKP hierarchy.

These results have lead to the **question**:

What is the most general relationship between the KdV and BKP hierarchies?

The answer to this question was recently given by the author. Namely, it was proved that *any* tau-function of KdV solves the BKP hierarchy:

Theorem 1. *For any KdV tau-function*

$$\tau_{BKP}(t) = \tau_{KdV}(t/2)$$

is a tau-function of the BKP hierarchy.

The proof of this theorem is based on the comparison of the corresponding Hirota bilinear identities. It is easy to see that the converse statement is false, that is, the KdV hierarchy is a certain reduction of the BKP hierarchy. There are several different ways to relate KdV and BKP hierarchies. In particular, already in the 80s Date, Jimbo, Kashiwara, and Miwa had described the identification of KdV hierarchy with the 4-reduction of BKP. Our reduction is much simpler than any of the previously known relations and provides a new, fundamental relation between two basic integrable hierarchies of solitonic type.

This result makes the Schur Q-functions a natural basis for an expansion of the KdV tau-functions. Such expansion can help to find new properties of the KdV tau-functions. For instance, the Schur Q-functions expansions of the Kontsevich–Witten and Brézin–Gross–Witten tau-functions have a special form: they describe so-called hypergeometric BKP tau-functions. This class of tau-functions, introduced by Orlov, is known to be related to an interesting class of the enumerative geometry invariants, namely, the spin Hurwitz numbers. Therefore, the identification of the KdV tau-functions with the solutions of the BKP hierarchy leads to the new, unexpected identification between two different classes of the enumerative geometry invariants, the intersection numbers on the moduli spaces and the spin Hurwitz numbers. In particular, the Brézin–Gross–Witten tau-function can be identified with the generating function of a spin version of dessins d'enfants.

We expect that the identification of the KdV tau-functions with the solutions of the BKP hierarchy will lead us to many new results in enumerative geometry and mathematical physics.

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Flexibility for Del Pezzo Cones

Jaehyun Kim
Graduate student member (POSTECH)

Hello, I'm Jaehyun Kim studying algebraic geometry at the Department of Mathematics, POSTECH and a student research assistant at IBS-CGP. It is a great honor to have had a chance to introduce my research in CGP Walk.

I would like to summarize the result in my recent paper and talk about some feelings I had during the research.

I am currenlty studying in pure geometric topic called cylindricity or cylindrical covering of projective varieties, especially for two dimensional Fano varieties called del Pezzo surfaces. Although it may not be familiar, ample polar cylindricity is one of an interesting topic which is equivalent to the existence of nontrival additive group actions on the corresponding affine cones. On the same note, the existence of certain cylindrical covering on the underlying projective variety ensures a significant property of the cone, called flexibility. As an equivalent condition of infinitely transitive action of special automorphism group, this flexibility is also an attractive topic in affine geometry.

Historically, there have been many attempt to find constructions and examples of flexible variety. In particular, for del Pezzo cones, there are some remarkable results as we move toward lower degrees for underlying surfaces. In terms of toric variety, the cones over del Pezzo surfaces of degrees at least 6 are all flexible[*]. And for degree 5[†], 4[‡] and recently even for 3[§], their flexibilities were also proved by the cylindrical coverings on del Pezzo surfaces. In fact for cubic surfaces, a more weaken version, called generic flexibility was verified. In succession to these methodology, we could show that the generic flexibility of affine cones over smooth del Pezzo surfaces of degree 2.

Now, to describe our main statement and its proof in detail, we should present some elementary definitions and properties. For convenience we will divide them into two parts, one is something affine and the other is that of projective geometry.

* Flag varieties, toric varieties, and suspensions : three examples of infinite transitivity by I. Arzhantset, M. Zaidenberg, and K. Kuyumzhiyan

† Flexibility of affine cones over del Pezzo surfaces of degree 4 and 5 by A. Perepechko

‡ Flexible affine cones over del Pezzo surfaces of degree 4 by J. Park and J. Won

§ Affine cones over cubic surfaces are flexible in codimension one by A. Perepechko

Affine Notions

A smooth point in an affine variety is called flexible if the tangent space at the point is spanned by the tangent vectors to the orbits of one parameter unipotent subgroups of the automorphism group of the variety. Thus, an affine variety is called *flexible* if each smooth point of the variety is flexible. And for the special automorphism group denoted by $SAut$ which is the subgroup of the automorphism group of the affine variety generated by all one parameter unipotent subgroups, it is said to act infinitely transitively on a set S if it acts transitively on the set of m-tuples of pairwise distinct points in S for any positive integer m. At this point, it is worthwhile to define the *generic flexibility* of affine varieties. An affine variety is called flexible in codimension r if there exists an open orbit of the $SAut$-action whose complement has codimension at least r. Then it is well known that an affine variety of dimension at least two is flexible if and only if its special automorphism group acts infinitely transitively on the smooth locus of the affine variety[*].

Projective Notions

A cylinder in a projective variety is a subset isomorphic to $A^1 \times Z$ for some affine variety Z. In particular, if the complement of the cylinder is given by the support of an effective $\mathbb{Q}$-divisor which is $\mathbb{Q}$-linearly equivalent to an ample divisor H, then it is also said to be H-polar. Generally, for the collection $U = \{U_a\}$ of open affine subsets of the normal projective variety, a subset A of the variety is said to be U-invariant if the intersection $U_a \cap A$ is $SAut(U_a)$-invariant for every open affine subset U_a in U, where $SAut(U_a)$ is the special automorphism group of U_a. The collection U is said to be *transversal* if the union of U_a's in U does not admit any nontrivial U-invariant subset. Moreover, the collection U is said to be *H-complete* if there exists no $\mathbb{Q}$-divisor equivalent to the ample divisor H whose support is contained in the complement of the union of U_a's in U.

As mentioned above, there are some connections between these two worlds as following. For a normal projective variety Y with an ample polarization H, its generalized cone $X = \text{Spec}(\bigoplus_{m\geq 0} \mathrm{H}^0(Y, \mathcal{O}_Y(mH)))$ admits a nontrivial unipotent group action if and only if Y has an H-polar cylinder[†]. Furthermore, if there exists a transversal covering (resp. H-complete collection) of H-polar cylinders on Y, then X is flexible (resp. flexible in codimension one)[‡§**]. From this point of view, we showed the generic flexibility of affine cones over smooth del Pezzo surfaces of degree 2. We have made a list of known examples of ample polar cylinders so far and we found another twinlike for one of each polarization to construct a transversal H-complete collection of polar cylinders so that we could prove the generic flexibility for almost all polarizations on the surfaces.

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[*] Flexible varieties and automorphism groups by I. Arzhantsev, H. Flenner, S. Kaliman, F. Kutzschebauch, M. Zaidenberg

[†] G_a-actions on affine cones by T. Kishimoto, Yu. Prokhorov, and M. Zaidenberg

[‡] Flexibility of affine cones over del Pezzo surfaces of degree 4 and 5 by A. Perepechko

[§] Flexible affine cones over del Pezzo surfaces of degee 4 by J. Park and J. Won

[**] Affine cones over cubic surfaces are flexible in codimension one by A. Perepechko

At first, we had tried to find any criterion of ample polar cylindricity for smooth del Pezzo surfaces of degree 2. As an attempt to expand the existing result for cubic surfaces*, we wanted to find a geometric or numerical characterization for ample polar cylindricity. Although we could attain only some partial answers, the various experiences have helped a lot in the current research. But also, it was fortunate to see other valuable research such as the generic flexibility for affine cones over cubic surfaces.

Now, I would like to take a moment and thank other colleagues, friends and family for their constant encouragement throughout my research. Especially, I would like to express my sincere appreciation to my advising professor for his warm and heartfelt academic advices. "Thanks to you all, I was able to continue in my research even in the midst of difficulties."

In the future, I would like to learn more about the fundamental tools of algebraic geometry and explore the Fano variety in depth as my lifelong research topic. I know that it might be difficult to suggest a new direction, but I will devote myself to this field with an open mind that can make mathematical flower bloom everywhere. I hope I will be able to see much more beautiful works in this field and make some contributions as well.

* Affine cones over smooth cubic surfaces by I. Cheltsov, J. Park, and J. Won

Surfaces in 4-manifolds and smooth 4-dimensional Poincaré conjecture

Seungwon Kim
Research member since September 2019

One of the most important open problems in low-dimensional topology is the Poincaré conjecture, which states that any homotopy n-sphere in some category (topological manifolds or smooth manifolds) is isomorphic in the chosen category (homeomorphic or diffeomorphic) to the standard n-sphere. In the topological category, the conjecture is true for every finite dimensions. The conjecture was solved by many famous mathematicians such as Stephen Smale [16] for dimensions greater than equal to 5, Michael Freedman [1] for dimension 4, and Grigori Perelman [13, 14, 15] for dimension 3. However, the situation is quite different for the smooth category. For dimensions less than equal to three, topological Poincaré conjecture implies the smooth Poincaré conjecture. However, in higher dimensions, John Milnor [12] found the exotic 7-spheres, the smooth manifolds which are homeomorphic to the 7-sphere but not diffemorphic to the standard 7-sphere. After this discovery, Kervaire and Milnor [5] built the algebraic framework to solve the smooth Poincaré conjecture for dimensions greater than equal to 5. However, this algebraic frame work does not work in dimension 4. In fact, the smooth Poincaré conjecture in dimension 4 is widely open. In dimension 4, there exist several potential counterexamples of the smooth Poincaré conjecture but only few of them were known to be diffeomorphic to the standard 4-sphere. In this research highlight, we will talk about one of the methods to produce such examples.

In 3-manifold theory, Dehn surgery along a knot plays a key role since every closed orientable 3-manifold can be obtained from Dehn surgery on S^3. In dimension 4, there is an analogous surgery called the Gluck surgery (or sometimes called the Gluck twist), which is a surgery operation along a 2-knot, an embedded (a knotted) sphere in S^4. Unlike dimension three, Gluck surgery along a 2-knot in S^4 can only produce homotopy 4-spheres [3]. It is conjectured that Gluck surgery may produce a counterexample to the 4-dimensional smooth Poincaré conjecture, but many such examples are known to be standard, such as Gluck surgery along ribbon 2-knots and twist-spun 2-knots. However, it is still unknown whether Gluck surgery along every 2-knot in S^4 gives back S^4 or not. There are several ways to study the Gluck twist. Here, we introduce one interesting method using surfaces in complex projective plane.

Consider the complex projective plane $\mathbb{CP}^2$. There is a complex projective line $\mathbb{CP}^1$ standardly embedded in $\mathbb{CP}^2$. A *unit sphere* is a 2-knot in $\mathbb{CP}^2$ which intersects $\mathbb{CP}^1$ transversely in one point. We can produce a non-standard embedding of 2-sphere which is unit by pairwise connected summing 2-knot pairs (S^4, F) and $(\mathbb{CP}^2, \mathbb{CP}^1)$, which is denoted by $(\mathbb{CP}^2, F\#\mathbb{CP}^1)$. Then Melvin [11] showed the following theorem, which allows us to study the Gluck surgery problem in S^4 as a surface classification problem in $\mathbb{CP}^2$.

Theorem 0.1. [11] *Gluck surgery along a 2-knot F in* S^4 *yields* S^4 *back if and only if the pair* $(\mathbb{CP}^2, F\#\mathbb{CP}^1)$ *is diffeomorphic to* $(\mathbb{CP}^2, \mathbb{CP}^1)$.

However, to do this classification, we need a method to study surfaces in general 4-manifolds.

In dimension three, we usually study knots via diagrams. There are infinite numbers of different diagrams which represent a given knot, so we need the Reidemeister moves to relate all such different diagrams. In dimension four, particularly S^4, there are several ways to relate possible representations of knotted surfaces, such as Roseman moves for broken surface diagrams, and Yoshikawa moves for banded unlink diagrams. For general 4-manifolds, Hughes, Miller, and I studied knotted surfaces represented by banded unlinks in a Kirby diagram $(\mathcal{K}, L, v)$ where $\mathcal{K}$ is a Kirby diagram which represent the 4-manifold, L is an unlink which bound mutually disjoint disks in 4-dimensional 1-handlebody, and v is a set of bands attached to L.

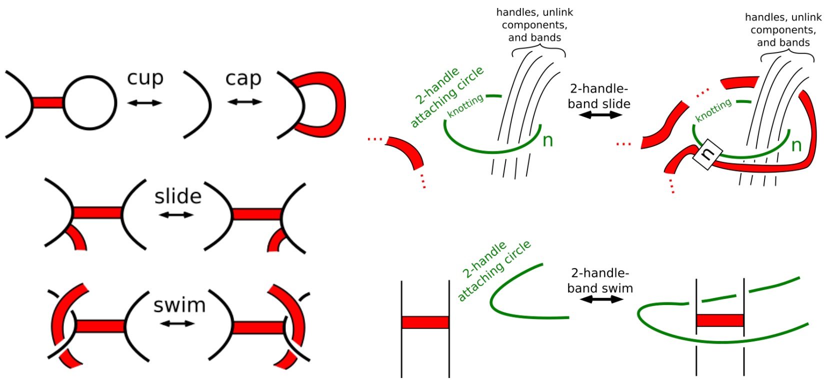

Figure 1. Band moves

Further, we introduced two new moves on banded unlinks in a Kirby diagram called the 2-handle-band swim and the 2-handle-band slide, and showed the following:

Theorem 0.2. [4] *Let* X^4 *be a smooth 4-manifold with Kirby diagram* $\mathcal{K}$. *Then every embedded surfaces in* X^4*can be represented by banded unlink diagram. Furthermore, let* Σ *and* Σ' *be embedded surfaces in* X^4 *and* $(\mathcal{K}, L, v)$ *and* $(\mathcal{K}, L', v')$ *be banded unlink diagrams for* Σ *and* Σ' *respectively. Then* Σ *and* Σ' *are isotopic if and only if* $(\mathcal{K}, L, v)$ *can be transformed into* $(\mathcal{K}, L', v')$ *by a finite sequence of band moves (See Figure 1).*

In 2012, Gay and Kirby [2] introduced a trisection of a 4-manifold, which is a 4-dimensional analogue of a Heegaard decomposition of a 3-manifold. After Gay and Kirby, Meier and Zupan [9, 10] introduced a *bridge trisection* of knotted surfaces in a 4-manifold. The theory of trisection gives a new way to understand 4-manifolds and knotted surfaces. For example, Meier, Schirmer and Zupan [8] studied the Generalized property R conjecture, and Lambert-Cole [7] gave a new proof of the Thom conjecture. To further study 4-manifolds and knotted surfaces theory, one potentially powerful tool involves translating them in terms of trisections and bridge trisections.

Meier and Zupan used Yoshikawa moves to show that every bridge trisection in S^4 can be related by a sequence of isotopies of each tangle, perturbations, and deperturbations. They further developed bridge trisection theory to an arbitrary 4-manifold and conjectured that a similar result holds in general. Hughes, Miller and I proved this conjecture in the affirmative.

Theorem 0.3. [4] *Let S and S' be surfaces in bridge position with respect to a trisection of a 4-manifold X^4. Suppose that S is isotopic to S'. Then S can be taken to S' by a sequence of perturbations and deperturbations, followed by a isotopies of trivial tangles in each 4-dimensional handlebody of a trisection.*

Now we can use above theorems to study the Gluck surgery problem in terms of surfaces in $\mathbb{CP}^2$. In [4], we showed several interesting classes of unit spheres in $\mathbb{CP}^2$ are isotopic to the standard $\mathbb{CP}^1$. Note that being isotopic is stronger than being pairwise diffeomorphic.

Theorem 0.4. [4] *Let $(\mathbb{CP}^2, F\#\mathbb{CP}^1)$ be a connected sum of pairs (S^4, F) and $(\mathbb{CP}^2, \mathbb{CP}^1)$, where F is a 2-knot. $(\mathbb{CP}^2, F\#\mathbb{CP}^1)$ is isotopic to $(\mathbb{CP}^2, \mathbb{CP}^1)$ whenever F is one of the following:*

- *ribbon 2-knots,*
- *twist spun 2-knots,*
- *Nash-Stipsicz 2-knots,*
- *2-knots which are 0-concordant to one of knotted spheres in the above list.*

Note that above lists of 2-knots are already known to have trivial Gluck twists. In [4], we also studied satellite 2-knots and proved that under certain assumptions, they are isotopic to the standard $\mathbb{CP}^1$. After this work, in [6], I applied the banded unlink diagram method and Kirby calculus to show that more general satellite 2-knots have trivial Gluck twists.

Theorem 0.5. [6] *Let K be a satellite 2-knot in S^4 of companion C with pattern (P, V). Suppose that the Gluck twist of S^4 along P is trivial. Then the following hold:*

(1) *If $[P] \in H_2(V) \simeq \mathbb{Z}$ is even, then the Gluck twist of S^4 along K is diffeomorphic to S^4.*
(2) *If $[P] \in H_2(V) \simeq \mathbb{Z}$ is odd and the Gluck twist of S^4 along C is trivial, then the Gluck twist of S^4 along K is diffeomorphic to S^4.*

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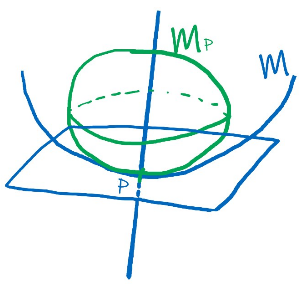
Infinitesimal symmetries of weakly pseudoconvex manifolds

Shinyoung Kim
Research member since September 2019

When we study manifolds, it is interesting to consider two point of view: one is a local geometry and the other one is a global geometry. As a very basic example; maybe as a level of undergraduate student, we can consider a smooth compact surface in 3-dimensional Euclidian space. Thinking a local chart of the compact surface, taking a bases, calculating Jacobians between two charts and thinking tangent vectors and a normal directional vector seems very local based on analytic calculation. On the other hands, thinking curves on the surface and geodesic flows between two points, giving an orientation on the surface, calculating the total curvature and Euler characteristic, thinking covering maps are relatively global and related to somehow different point of view, algebraic or topological.

Before I start the project "Infinitesimal symmetries of weakly pseudoconvex manifolds" with Prof. Martin Kolář, I had been focused on the local structures of non-homogeneous horospherical varieties with Picard number one which are quasi-homogeous manifolds ([a], [b]), in particular, I found gradation on its infinitesimal automorphism leveled by rational numbers. That is known that it is possible to have that kind of rational gradation but examples are not well-known among all complex manifolds, even including CR-manifolds. When I met Martin, we agreed at this point and he had an example. We agreed to find more examples and I also hoped to know the reasons of rational leveling including all the cases that I know. That was a strong personal motivation, but shortly after, I notice that it might be a quixotic thinking even both rational leveling are from non-homogeneous conditions. That happened because I did not have any background of CR geometry at all and I only had focused on local behavior, specially, the Lie algebras of infinitesimal automorphisms which can be expressed as graded Lie algebras for some purpose. I can say, that has similar distance between Dynkin diagram for root system and Dynkin graphs for singularities if there is a possible metric to compare.



On a manifold with smooth boundary, the Levi-problem is finding a holomorphic function which cannot pass to a boundary point. Locally, positive definite Levi form guarantees the solution of the Levi-problem. A following question is existence of boundary solution when the Levi form is semi-positive definite. Since Levi form control the local regularity of inhomogenous Cauchy-Riemann operator, we denote it ∂-bar, the problem is related to so-called ∂-bar Neumann problem.

When the boundary of manifold is equivalent to a strongly pseudoconvex hypersurface in n dimnesional complex vector space, by the classical work of Poincaré, Levi. Cartan, Tannaka, Chern, Moser, Fefferman and many others, the study of invariants and symmetries were essentially important. A Cauchy-Riemann invariant of the weakly pseudoconvex manifold of lowest dimension with integer valued type, were initially studied by J. J. Kohn [c]. For characterizing subellipticity of the ∂-bar Neumann, more refined local invariants of weakly pseudoconvex manifolds had crucial role in the work of D. Catlin [d][e].

A concrete question given by D. Zaitsev was what possible symmetries of weakly psedoconvex manifolds of Catlin multi-type is. A pseudoconvex hypersurface, which is locally given by a weighted homogeneous polynomial in complex tangential variables, is called Catlin multi-type if the polynomial has multi-type weights. The corresponding (local) polynomial is called model which is given by the weighted homogeneous polynomial with multi-type weights. M. Kolář, F. Meylan and D. Zaitsev [2] showed, using generalized Chern-Morser operator, that the Lie algebra of infinitesimal automorphism of polynomial model admits the weighted gradation of depth 1 with non-integer levels; negative multi-type weights and integer combination of multi-type weights for which holomorphic nondegeneracy of the hypersurface. Based on their work, we showed ([1]) that the Lie algebra of infinitesimal automorphism of holomorphic nondegenerate weakly psedoconvex hypersurface with non-vanishing depth 1, then it admits a gradation with non-integer levels of 1/2 and negative multi-type weights. Moreover, the 0 level is generated by weighted Euler field and a subalgebra of a unitary Lie algebra.

Theorem 1.2 Assume that M is pseudo convex in a neighborhood of p with a homomorphically non degenerate model hypersurface M_P, given by (1.2) and non vanishing $\mathfrak{g} = aut(M_P,0)$ of M_P admits the weighted grading given by

$$\mathfrak{g} = \mathfrak{g}_{-1} \oplus \bigoplus_{j=1}^{n} \mathfrak{g}_{-\mu_j} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_{\frac{1}{2}} \oplus \mathfrak{g}_1,$$

where $\mathfrak{g}_0$ is generated by the Euler field and a sub algebra of $\mathfrak{u}(n)$.

Example. Let M be a hypersurface in $\mathbb{C}^N$ given by Im$w = P(z,\bar{z})$, where

$$P(z,\bar{z}) = A_1|z_1|^{2k_{11}} + A_2|z_1|^{2k_{12}}|z_2|^{2k_{22}} + \ldots + A_n|z_1|^{2k_{1n}}|z_2|^{2k_{2n}}\ldots|z_n|^{2k_{nn}}.$$

Any element of $\mathfrak{g}_0$, which is not a Euler field, is of the form

$$X = \sum_{j=1}^{n} A_j z_j \partial_{z_j},$$

where A_j are arbitrary purely imaginary numbers. Since the model is sums of squares of degree > 2, we obtain $\dim \mathrm{aut}(M,0) = n + 3$ with the first symmetry, the Euler field (the second one) and third symmetry.

As a consequence, we see that the automorphism of holomorphic nondegenerate weakly psedoconvex hypersurface with non-vanishing depth 1 determined by its jet of order 2. When the polynomial model is sum of squares which satisfy balanced condition, the automorphism has non-vanishing depth 1 and the negative non-integer levels has only -1/2 value. That is much simpler than general cases. Since only integer level 1, 0 and -1 are appears when the polynomial is homogeneous, in this sum of square cases, we can say the rational leveling comes from the multi-type weights. A basic example is a homogeneous sphere model which is an integer multiple of a sphere. For the proofs, it is not an overstatement that we only use linear algebras and the definition of holomorphic vector fields. We started with sum of squares models and after checked same properties for general cases. It was interesting work rather than difficult. Jordan normal form and the idea of pigeon hole principle were also used.

By M_S we will denote a sum of squares model, given by

$$\text{Im}\,(w) = \sum_{j=1}^{k} |P_j(z)|^2,$$

where P_j are weighted homogeneous holomorphic polynomials of weighted degree $\frac{1}{2}$ with respect to the multi type weights.

My next research plan is looking things more globally. Local equivalence problem related to this project maybe one way for this purpose, mentioned by M. Kolář during the project. But since I mainly have been focused on the woks of quasi homogenous complex projective manifolds ([f]-1,2,3), it seems better me to push in the related projects for a meantime. There are two ongoing project that I recently focusing on; one is about minimal rational curves on complete symmetric varieties with N. Perrin and M. Brion and the other one is about characterization of horospherical varieties with Picard number one with J. Hong. These two are deeply related to my first aimed project started with J. Hong which takes a bit more time than we expected. In addition, I would like to express gratitude through this article, to IBS-CGP for their support, to my colleagues whom are always kind and friendly encourage me, specially, for my visiting in 2017/2018 to Brno in Czech, and also to my lovely family.

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Ulrich bundles on Fano varieties

Kyoung-Seog Lee
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1. Ulrich vector bundles on algebraic varieties

Vector bundles on an algebraic variety are important objects containing lots of information about the variety. The theory of vector bundles on algebraic variety has a long history and plays important roles in many branches of mathematics. For example, a fundamental theorem of Narasimhan and Seshadri says that having a stable bundle on a smooth projective curve is equivalent to having a unitary representation of the fundamental group of the curve. This result provides bridges between algebraic geometry, differential geometry and topology, and is later generalized to higher-dimensional complex manifolds by Donaldson, Uhlenbeck and Yau. Vector bundles on algebraic varieties also play important roles in gauge theory. For example, Donaldson used the theory of vector bundles to prove that a Dolgachev surface is homeomorphic but not diffeomorphic to a rational elliptic surface obtained by blowing up 9 points at P^2. There are many more important results in the theory of vector bundles and it is still a very active area of research.

Topological type of a vector bundle on a smooth projective curve is determined by its rank and degree. For higher-dimensional algebraic varieties, one needs more data (e.g. Chern classes) to determine topological types of vector bundles on them and it seems to be impossible to study all vector bundles on a given higher-dimensional algebraic variety. Therefore, it is natural to study special vector bundles on a given higher-dimensional algebraic variety. People have studied instanton bundles which are important objects in gauge theory and ACM bundles whose twist do not have any nonzero intermediate cohomology group. Ulrich bundles are special ACM bundles having several nice properties. To be precise, for a smooth projective variety X of dimension d embedded in the complex projective space P^N, a vector bundle E on X is called Ulrich if the cohomology groups of its twisted bundle satisfy $H^i(X, E(-k)) = 0$ for all $0 \leq i \leq d$ and $1 \leq k \leq d$. Ulrich bundles form interesting classes of vector bundles on higher-dimensional algebraic varieties. Recently, there have been lots of works studying the existence and the moduli of Ulrich bundles. For example, Eisenbud and Schreyer conjectured that every projective variety will admit an Ulrich sheaf and their conjecture have been verified for several algebraic varieties, e.g. complete intersection varieties, del Pezzo surfaces, K3 surfaces, abelian surfaces, and non-special surfaces with $p_g = q = 0$ etc.

Figure 1. Poster of "Vector Bundles on Algebraic Varieties", 2017

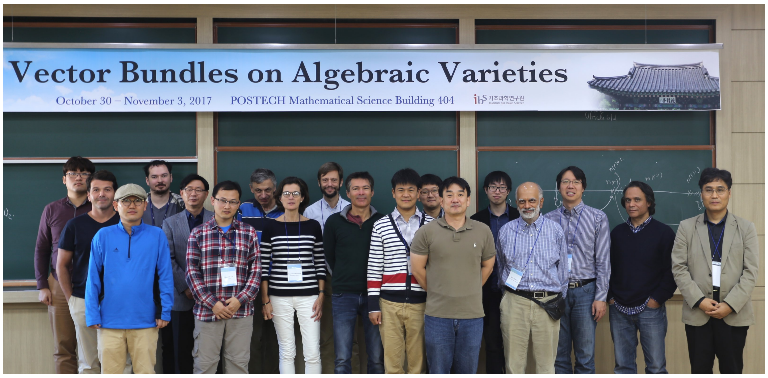

Figure 2. Group photo of "Vector Bundles on Algebraic Varieties", 2017

2. Equivariant Ulrich bundles on homogeneous varieties

Equivariant Ulrich bundles on some rational homogeneous varieties were studied by many researchers. To be more precise, Costa and Miró-Roig, and Coskun et al. classified irreducible equivariant Ulrich bundles on Grassmannians, and partial flag varieties of algebraic groups of type A, respectively. Then Fonarev classified irreducible equivariant Ulrich bundles on isotropic Grassmannians of algebraic groups associated to other classical groups of type B, C, and D.

We studied irreducible equivariant Ulrich bundles on rational homogeneous varieties with Picard number 1 of exceptional algebraic groups. We proved that the only rational homogeneous varieties with Picard number 1 of the exceptional algebraic groups admitting irreducible equivariant Ulrich bundles are the (16-dimensional) Cayley plane E_6/P_1 and the (33-dimensional) E_7-adjoint variety E_7/P_1. The idea of the proof is as follows. From the Borel–Weil–Bott theorem, we can obtain a criterion for an irreducible equivariant vector bundle on a rational homogeneous variety G/P to be Ulrich. We compute and analyze the set of singular values case by case: Hermitian symmetric spaces of compact type, adjoint varieties, simply laced cases of type A, D, E, etc. From the above result, we completed the classification of irreducible equivariant Ulrich bundles on rational homogeneous variety with Picard number 1. We can also see that a general hyperplane section F_4/P_4 of the Cayley plane E_6/P_1 has an equivariant but non-irreducible Ulrich bundle.

3. Moduli spaces of Ulrich bundles on Fano threefolds

Many researchers have studied ACM bundles on surfaces and Fano 3-folds. For example, Kuznetsov studied instanton bundles on some Fano 3-folds via their bounded derived categories of coherent sheaves. Using derived categories, Lahoz, Macri and Stellari studied moduli spaces of Ulrich bundles on cubic 3-folds and 4-folds, and Cho, Kim and Lee studied Ulrich bundles on intersections of two 4-dimensional quadrics.

We described moduli spaces of Ulrich bundles on V_5 which is the unique smooth Fano 3-fold of Picard number 1, degree 5 and index 2. The variety V_5 is a very interesting Fano 3-fold which enjoys many beautiful geometric and topological properties. It is well-known that V_5 is a generic codimension 3 linear section of the Grassmannian $Gr(2,5)$ of 2-dimensional subspaces in a 5-dimensional complex vector space, so that we can explicitly compute the cohomology groups $H^i(V_5, U(j))$ and $H^i(V_5, Q^*(j))$ using the Borel–Weil–Bott theorem and the Koszul resolution, or the Riemann–Roch theorem.

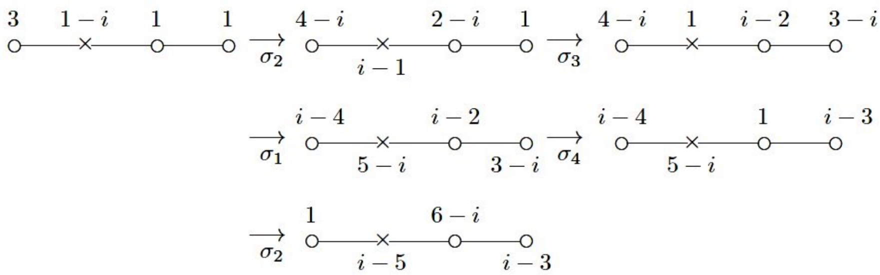

Figure 3. Cohomology computation of Ulrich bundle $Sym^2(U^*)$ on $Gr(2,5)$

Vector bundles on V_5 were studied by Arrondo and Costa, Faenzi, and Kuznetsov. The bounded derived category $D(V_5)$ of coherent sheaves on V_5 was studied by Orlov where he proved that there are full exceptional collections in $D(V_5)$. From one of the exceptional collections and Bondal's work, we see that the derived category of the finitely generated modules of the path algebra associated to the Kronecker quiver Γ with 2 vertices and 3 arrows is embedded in $D(V_5)$.

Using this semiorthogonal decomposition of the derived category of V_5, we obtain the following result: For any $r \geq 2$, there is a nontrivial $(r^2 + 1)$-dimensional family of stable Ulrich bundles of rank r on V_5. Moreover, the moduli space of stable Ulrich bundles of rank r on V_5 is isomorphic to a smooth open subset of the moduli space of stable quiver representations of the quiver Γ with dimension vector (r, r). In particular, V_5 is of Ulrich-wild representation type.

Our work motivated other mathematical works. When $r = 2$, the moduli space of semistable quiver representations of the quiver Γ with dimension vector (2,2) is isomorphic to P^5. Therefore, the moduli space of Ulrich bundles on V_5 is isomorphic to an open subset of P^5. Qin studied modular compactification of the moduli space of Ulrich bundles of rank 2 by considering instanton sheaves and proved that the compactification is isomorphic to P^5. He proved that a similar modular compactification of the moduli space of rank 2 Ulrich bundles on the intersection of two 4-dimensional quadrics is isomorphic to the moduli space of semistable vector bundles of rank 2, degree 0 on the associated genus 2 curve using the work of Cho–Kim–Lee.

There are many open questions about Ulrich bundles on varieties and many people are working on this subject. Especially, the conjecture of Eisenbud and Schreyer seems to be out of reach at this moment. It will be nice to construct Ulrich bundles on certain special classes of varieties like toric varieties, spherical varieties, etc. And studying moduli spaces of vector bundles is also an important remaining task. We think our works can be meaningful steps toward the answers of these questions.

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Interviews

Youngjin Bae

Sangwook Lee

Saswati Dhara

Youngjin Bae

Research member from March 2013 to November 2017

How is your life / work after CGP?

After working at CGP, luckily, I was selected for a JSPS postdoc position which was funded by the Japanese government. This was a two years position in the Research Institute for Mathematical Sciences located in Kyoto University. Looking back to the two years in Japan, it was the most pleasant moment in my life. I truly liked the discussions with colleagues and my host professor Kaoru Ono, as well as the culture, landscape, and nature of Kyoto. Especially the sunset near Kamogawa river, which I used to visit and enjoy will always be in my heart.

In 2019, I had to come back to Korea for my next postdoc position at KIAS. Around that time, COVID-19 began to spread, so every activity including discussion with nearby researchers, meeting and making friends were restricted. I guess because of COVID-19 many people felt constrained and low in energy. To be honest, so did I.

Now I have started a new life at Incheon National University. Still the pandemic is going on, but I am slowly recovering and rebalancing to a positive state of mind and body. Even though I already spent one year in Incheon, still many things are not familiar including preparing and recording the lectures, online meeting with colleagues, and online tutoring for students. I really hope the pandemic ends soon, so I can have a face-to-face meeting with colleagues and students.

What made you decide to be a mathematician?

Truth be told, I wanted to be a high school teacher in mathematics. I did graduate from the college of education. At the time of graduation, I asked myself "Are you really ready to become a teacher?" In front of future students I would have to persuade them by saying things like "Doing math is meaningful and enjoyable." But I was not that confident to say such words. All I had done relating to math during my undergraduate was learning and absorbing results efficiently. Frankly, I had never DID mathematics. I thought it was not good for myself and also for my future students. So I decided to DO mathematics more in depth. This is the starting point of my research career.

I am not good at music. I do not know anything on the laws of harmony and scale. If I see a musical score, it is just a sequence of meaningless musical notes on paper. But to someone who knows how to read and interpret the musical notes it could become thrilling music.

I feel that the same works for mathematics. Math books and articles are full of formulas and equations which are boned with complicated logics. If you are trained to understand and savor those strange words and equations, then it certainly shows the beauty and harmony of the inner structure of mathematics. Understanding and seeking the hidden structures in the math world is my strong motivation for doing mathematics.

**What is your current math-related interest?
Please tell us about your research.**

Currently, I am interested in a geometric realization of algebraic structure. More precisely, I want to construct 2-dimensional Lagrangian fillings of 1-dimensional Legendrian links which realize seeds in the cluster structure of the algebraic side.

There are several ways and directions of doing mathematics. One criterion to distinguish mathematicians could be "abstraction vs concretization". Some prefer to work with abstract notions, while others feel comfortable with specific and concrete examples. I am surely the latter case. My mathematical toys, Legendrian graphs and links, are very visible and relatively easy to describe. But there are also subtle aspects in the sense of classification, and this subtlety is deeply related to the heart of symplectic geometry and contact topology.
In summary, my research topic is to understand general slogans in symplectic geometry via Legendrians.

**What are you interested in recently?
Please share something about you.**

Climbing! I could imagine my life without mathematics, but I think I cannot live without nature and rock climbing. I am not that talented in climbing but certainly it became a part of my identity.

I usually go to a nearby indoor climbing gym for bouldering once or twice a week, and visit an outdoor rock site if the weather permits on the weekend. The most attractive points of rock climbing or bouldering is that you can see your grade of climbing in a very clear and direct way, and every level could find their own route to a continuous challenge. I also really like the feeling of immersion in climbing. Even with much worries or burdens I have on my mind, I could get out of them when I am facing the walls or rocks when I climbing.

We would like to hear about your dream or future plans. Do you have a role model or a philosophy of life? :)

My future plan is relatively simple. First, enjoy math and climb as much as I can. Second, upgrade the level of math researcher and climbing at least until I reach the age of 60. I really hope I can accomplish this. All the people who improve skills in their work area are my mentors, especially older people.

The main problem or topic of my life in the last decade was about "passion and obsession", and yet they are still unsolved. For me, it is hard to maintain passion without obsession. When I start to obsess about something like the outcome of the research, people I like, or a rock climbing route, I usually repeat myself with the words "The results are not mine. Man proposes, God dispose."

I don't think I am eligible to answer this question yet. I still regard myself as a young researcher and yet immature.

Is there anything you want to tell younger mathematicians?

If you are talented at math, no word for you, I just envy you. Otherwise, if you are someone who just loves to do math, then I have some words for you. Maintain your passion consistently and don't lose faith, continue in math like a hen incubating her eggs.

Sangwook Lee

Research member from April 2013 to March 2016

How is your life / work after CGP?

I worked at KIAS as a research fellow, and then joined Soongsil University as a faculty member last year. Because I was only at research positions before I came to university, many things (mainly including teaching students) are now totally new and interesting though I only could give online lectures so far. The biggest change after CGP however is that I became a father! Like doing mathematics, bringing up children has many aspects. Often it is painful and tough, but I also encounter a number of fascinating moments.

What made you decide to be a mathematician?

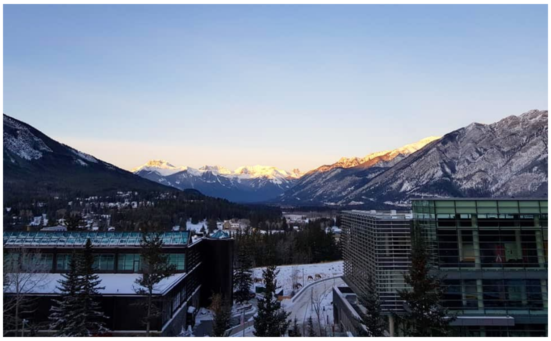
Mathematics does not reveal its beauty so easily, but sometimes I can see a little bit of it after many days of struggle. Any mathematician has his/her own home based on the creativity, in the realm of mathematics, however little or humble it looks like. It is worthwhile to discover such a new territory of mathematical world.

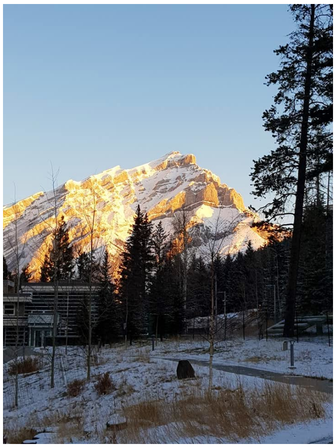
What is your current math-related interest? Please tell us about your research.

I have been working on Hochschild invariants of categories which appear in mirror symmetry. Basically there are two kinds of Hochschild invariants: one is homology and the other is cohomology. If I look at homology, I get to consider vector spaces with nondegenerate pairings which have various geometric origins. After a substantial investigation on such homology structures, I moved on and have worked on ring structures on Hochschild cohomology. If we consider group actions on manifolds or algebraic functions, then the structure becomes much more interesting and complicated, and I recently found how to interpret one of previously known concept(a.k.a twisted Jacobian algebras) in terms of matrix factorizations.

What are you interested in recently? Please share something about you.

I had chances to live in foreign countries for months as participating in thematic research programs. At those times I could spend some time to travel nearby regions. I recall beautiful streets in Gent, the atmosphere of Christmas market in Bonn, Niagara falls and tremendous mountains and lakes in Banff. I really miss all of them and hope the future when I can go there with my family again.





We would like to hear about your dream or future plans. Do you have a role model or a philosophy of life? :)

As a person, I want to be more trustworthy and responsible so that people around me might feel secure and comfortable.

As a professor, I get to think more on the meaning of teaching mathematics to students. However one thing is evident: I can provide students with more chances and opportunities in every aspect as long as my mathematical world has enough depth and width.

Is there anything you want to tell younger mathematicians?

I am certain that recent young researchers are really really excellent. I just hope that you may have assurances of their own excellences, not being intimidated by what other people are doing. On top of that, try to show yourself as a person whom people joyfully like to work with. It will help you proceeding to next stage in your career.

Saswati Dhara

Research member since October 2020

How is your life in CGP / Pohang / Korea?

I am an Indian. I travelled to Korea last October (during COVID pandemic) to start my postdoctoral career at IBS-CGP. Before my trip, I was worried a lot regarding my journey but at the end it was super smooth thanks to Genn Ia Kim and IBS-CGP for their kind help in everything.

Regarding myself, I am a nature lover. So, I had love at first sight when I came to CGP for the first time due to its location. It is full of natural sceneries; a calm and positive atmosphere creates a perfect ambience for research.

Now, coming to Pohang, it is a small and nice city. While the green mountains pour their blessings upon it, the sea touches its feet. A calm and beautiful river is also passing through the city which further adds to its charm. I must say, I have fallen in love with Pohang. About Korea, I have not interacted with many people. However, from my own experience people here are helpful.

What made you decide to be a mathematical physicist?

My father is a school teacher who teaches mathematics and physics. When I was a kid, I used to listen to stories from him. As I started to grow, he gave me some story books related to physics. That created an impact in my mind to know more about it. Since I was good at math, I always tried to understand the physics concepts using mathematical tools. During my PhD time, I found an opportunity to work in mathematical physics related topics which further created my interests in this field.

What is your current physics-related interest? Please tell us about your research.

My interest lies in understanding fascinating concepts of physics using the language of mathematics. During my PhD, I was working on the Chern-Simons theory of knot/link invariants and topological entanglement. In particular, I was involved in the computation of non-arborescent knot/link invariants.

Matrix model helps to obtain a class of natural solutions to integrable hierarchies which help to analyze some characteristic properties of two-dimensional gravity. Currently, I am working on topics related to a matrix model known as Brézin–Gross–Witten model.

What are you interested in recently? Please share something about you.

Due to the pandemic, I was only able to make one trip to Seoul in the last one year. In any case, my hobby is to enjoy the beauty of nature quietly sitting in a place with some background music. During the Korean summer, I used to go regularly near the Pohang River to spend some quality time. I always try to capture nature in my small camera. Here you find some glimpse of that.

We would like to hear about your dream or future plans. Do you have a role model or a philosophy of life? :)

For now, my idea is to enjoy the research as much as possible. In the future, I am interested in teaching physics. I have also thought about making a small garden surrounding my house. There are several people whom I admire. My father is one of them who taught me how to live life with honesty and full of self-respect. Next, I will come to my husband. Being a best friend, he supported me in every step including my decision to come to Korea. I would also like to give the credit to my PhD supervisor, Prof. Ramadevi who taught me many things in research. Finally, my philosophy of life is to live simply and happily.

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CGP Walk
2nd Issue | 2021

- Publisher — Yong-Geun Oh, IBS Center for Geometry and Physics
- Editor-in-chief — Jihun Park
- Editors — CGP Administration Team
- Published — December 2021

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