

IBS Center for Geometry and Physics

# CGP Walk

— *Beyond the horizon* —



The First Issue

2019 - 2020

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## Greetings : Director's Note

It has been already eight years since the founding of Center for Geometry and Physics (CGP) as one of the 9 inaugural research centers of the IBS, and as the only one in mathematics. When CGP was open, I had the vision of CGP as an ideal scientific research institute that fosters the disinterested pursuit of learning just like Institute for Advanced Study (IAS) in Princeton and Institut des Hautes Études Scientifiques (IHES) in Paris.



However I soon realized how ambitious this was:

I had to face the obstacles of no stable IBS blueprint, difficulty of recruiting, the requirement of preparing extensive annual report and the re-budgeting based thereon. I had not imagined these obstacles when I first joined in IBS thinking that, as the founding blueprint of the IBS center laid out, the center would be given such a freedom that IAS or IHES enjoy. However I not only had to survive as a researcher but also had to become a supervisor overseeing other people's performance in their research which was quite stressful. But I had to embrace and adjust to this reality. At that time, I decided to do my best in making CGP into a place of members' disinterested pursuit by doing my best in my research as a role model.

After all these, it is very satisfying to witness the CGP members sprouting the seeds of creation during the time of CGP and many CGP alumni making dents on the mathematical horizon. It may also be inevitable but is sometimes painful as a director and mentor to see some of them find doing mathematical research not the best option in their life paths. But I am confident that their experience in mathematical thinking will benefit the society as well as themselves in other ways.

I am very happy that the CGP's 8th year evaluation went smoothly. This has been possible by the group effort of Professor Jihun Park and CGP members, not only of the scientific research members but also of our devoted administrative staffs. I want to take this opportunity to thank all of CGP members for their devotion, and the CGP Scientific Advisory Committee members for their valuable advice and criticism. In the midst of this COVID-19 pandemic, we are meeting a new challenge of adapting to this new alien environment and turning this challenge into our opportunity to upgrade and excel. We shall overcome this challenge together.

## Background and Vision

The Center for Geometry and Physics (CGP) was founded in July 2012 as one of the first research centers of the Institute for Basic Science (IBS). The CGP originated in a government funded award, via IBS, to the research program of its director Yong-Geun Oh. This program aims to help establish and develop the emerging field of symplectic algebraic topology through a collaborative effort by experts in fields such as symplectic geometry, dynamical systems, algebraic geometry and mathematical physics. The Center is currently evolving into an international institution with a broader scope, focusing more generally on geometry and mathematical physics.

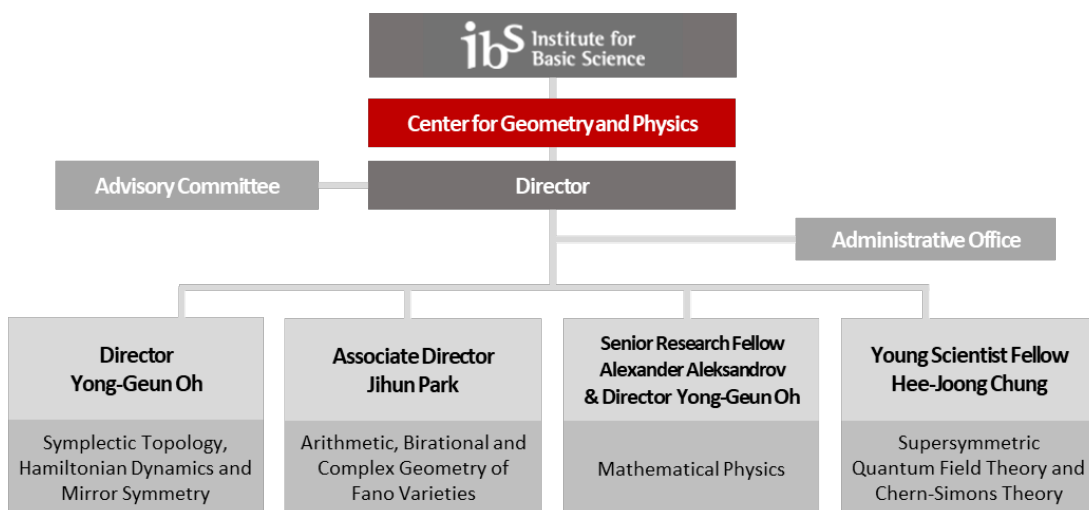
An ideal scientific research institute should be a place which fosters the disinterested pursuit of learning and the fundamental innovative thinking that advances the individual fields of inquiry of an intellectual community. The mission of the Center for Geometry and Physics (CGP) is to enable the research environment at the CGP to achieve this ideal. By now, the CGP has created such an atmosphere that ideas from mathematics and physics are naturally shared and interact. The CGP will ensure maintaining this unique research environment to serve the international community by

- Providing intellectual leadership and stewardship, guiding the development of relevant mathematics in fruitful directions by strengthening the interaction between geometry and physics,
- Playing the role of a physical nexus in Korea and beyond for the events which are the social glue of mathematical progress by hosting workshops, conferences, visitors, and so on, in order to lubricate the flow of ideas throughout the international community,
- Becoming as an incubator for young mathematicians, giving them the time and freedom to pursue ambitious and idiosyncratic research goals in a nurturant and enriching environment.

## Organization

One arching research theme of the CGP is to promote interaction between symplectic geometry, algebraic geometry and mathematical physics in the study of symplectic topology and homological mirror symmetry and their applications to theoretical physics.

The CGP is loosely organized into multi research groups, each of which comprises a senior scholar who leads the group and several researchers whose areas of expertise and interest overlap synergistically. The CGP also hosted a Young Scientist Fellow who is closely related with those of three main research groups.



## Research Groups

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### Symplectic Topology, Hamiltonian Dynamics and Mirror Symmetry

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**Team Leader: Yong-Geun Oh**

The current status of symplectic topology resembles that of classical topology in the middle of the twentieth century. Over time, a systematic algebraic language was developed to describe problems in classical topology. Similarly, a language for symplectic topology is emerging, but has yet to be fully developed. The development of this language is much more challenging both algebraically and analytically than in the case of classical topology. The relevant homological algebra of  $A_\infty$  structures is harder to implement in the geometric situation due to the analytical complications present in the study of pseudo-holomorphic curves or "instantons" in physical terms. Homological mirror symmetry concerns a certain duality between categories of symplectic manifolds and complex algebraic varieties. The symplectic side of the story involves an  $A_\infty$  category, called the Fukaya category, which is the categorified version of Lagrangian Floer homology theory. In the meantime, recent developments in the area of dynamical systems have revealed that the symplectic aspect of area preserving dynamics in two dimensions has the potential to further understanding of these systems in deep and important ways.

*Research members and their research themes:*

- ✦ **Hongtaek Jung** (Symplectic structures of Hitchin components and Anosov representations)
- ✦ **Sungkyung Kang** (Heegaard Floer theory, knot theory)
- ✦ **Jongmyeong Kim** (Homological mirror symmetry)
- ✦ **Seungwon Kim** (Topology and geometry)
- ✦ **Taesu Kim** (Homotopy theoretic aspects of symplectic geometry)
- ✦ **Myeonggi Kwon** (Contact and symplectic topology)
- ✦ **Eunjeng Lee** (Toric topology, Newton-Okounkov bodies, representation theory, and algebraic combinatorics)
- ✦ **Sangjin Lee** (Lagrangian foliations, Symplectic mapping class group, Fukaya category)
- ✦ **Yong-Geun Oh** (Symplectic topology, Hamiltonian dynamics and mirror symmetry)
- ✦ **Minkyung Song** (Geometric topology and related topics in group theory)
- ✦ **Yat-Hin Suen** (Complex geometry, Symplectic geometry, SYZ mirror symmetry, Homological mirror symmetry)

## Arithmetic, Birational and Complex Geometry of Fano Varieties

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### Team Leader: Jihun Park

Fano varieties are algebraic varieties whose anticanonical classes are ample. They are classical and fundamental varieties that play many significant roles in contemporary geometry. Verified or expected geometric and algebraic properties of Fano varieties have attracted attentions from many geometers and physicists. In spite of extensive studies on Fano varieties for more than one centuries, numerous features of Fano varieties are still shrouded in a veil of mist. Contemporary geometry however requires more comprehensive understanding of Fano varieties.

### *Research members and their research themes:*

- ✦ **Shinyoung Kim** (Complex geometry)
- ✦ **Jihun Park** (Arithmetic, birational and complex geometry of Fano varieties)
- ✦ **Jun Yong Park** (Arithmetic of the moduli of fibrations)
- ✦ **See-Hak Seong** (Algebraic geometry)
- ✦ **Yuto Yamamoto** (Tropical geometry)
- ✦ **Sungmin Yoo** (Complex geometry and geometric analysis)

## Mathematical Physics

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**Team Leader: Alexander Aleksandrov and Yong-Geun Oh**

The mathematical relevance and deep interconnections between theoretical physics and mathematics are well-established. This subject is universally appreciated for its integrative role and for being one of the most fruitful sources of new ideas, theories and methods, and have numerous powerful applications to problems in mathematics, in particular, of geometry and topology. In recent decades, there have been various developments in supersymmetric quantum field theories and string/M-theory. In this premise, matrix models, integrable systems, Chern-Simons gauge theory, Landau-Ginzburg theory and mirror symmetry, and topological quantum field theories are the main themes of research pursued in this group.

*Research members and their research themes:*

- ✦ **Alexander Aleksandrov** (Mathematical physics, random matrix models, integrable systems, enumerative geometry)
- ✦ **Anna Cepek** (Manifold topology)
- ✦ **Saswati Dhara** (Theoretical high energy physics, Chern-Simons theory in knot invariants, conformal field theory, topological field theory)
- ✦ **Damien Lejay** (Mathematical physics, vertex algebras, factorisation algebras, higher toposes)
- ✦ **Yifan Li** (Algebraic geometry, algebraic topology and mathematical physics)

## Supersymmetric Quantum Field Theory and Chern-Simons Theory

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**Team Leader: Hee-Joong Chung (Young Scientist Fellow)**

In recent decades, there have been various developments in supersymmetric quantum field theories in various dimensions. Among many interesting advances, the correspondence between  $d$ -dimensional non-supersymmetric theories and  $(6-d)$ -dimensional supersymmetric theories that is realized from the  $6d$   $N=(0,2)$  theory on the product space of a  $d$ -dimensional space and a  $6-d$  dimensional space has attracted attention both from physicists and from mathematicians. Such correspondences have been proposed for  $d=2, 3, 4$ , and they are also related to each other when 3-manifolds are boundaries of 4-manifolds or have Riemann surfaces as boundaries. When  $d=3$ , the correspondence involves the Chern-Simons theory. Many interesting aspects of the correspondences have been studied, and there are still many interesting problems. We would like to explore various aspects of the correspondences with focus on the case of  $d=3, 4$ . In addition, we would also like to explore other interesting topics in supersymmetric quantum field theories and topological quantum field theories.

*Research members and their research themes:*

- ◆ **Hee-Joong Chung** (Quantum field theory, String/M-theory, Physical mathematics)

## CGP Advisory Committee

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The CGP Advisory Committee consists of eight distinguished scholars from Korea and abroad. The committee meets once a year and provides advice and input on the operations of the Center.

The current members of the Advisory Committee are:

### **YunSung Choi**

Professor, POSTECH

### **Mikhail Kapranov**

Professor, Kavli IPMU\*

### **Ludmil Katzarkov**

Professor, University of Miami

Professor, Higher School of Economics

### **JongHae Keum**

President, Korean Mathematical Society

Professor, KIAS

### **John Morgan**

Founding Director, Simons Center for Geometry and Physics

### **Kyewon Koh Park**

Research Professor, Center for Mathematical Challenges, KIAS

### **Dong Youp Suh**

Professor, KAIST

### **Herman Verlinde**

Professor, Princeton University

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\* Kavli Institute for the Physics and Mathematics of the Universe

## Research Infrastructure



The Center for Geometry and Physics aims to provide a research environment in which new and original ideas are boldly proposed, tested and revised by means of scientific interactions and communication. By doing so, we hope that some of those ideas evolve into a mature form of truly new mathematics. Thus, the goal of the center is to become the birthplace of fundamentally new research areas in addition to carrying out those projects envisioned in its initial proposal.

## CGP Hall & Library

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The CGP Hall is the central location of the Center's academic and social activities.



The hall serves as the venue for most of the seminars, talks and teatimes hosted by the Center. Members and visitors often gather here in small groups for discussions, exchange of ideas or simply for relaxing. It also has several offices for visitors and some members.

The CGP library collection of 4,526 books that the Center has established in topics related to the research areas of the Center greatly complements the rich archive sources available to its members and visitors. The entire CGP library collection is housed at the CGP Hall, and is open to POSTECH members as well so that students and researchers at the Department of Mathematics can take advantage of the collection.



## Website, Video System, and Computing Facilities

The CGP website (<http://cgp.ibs.re.kr>) continues to provide schedules and information on events hosted not only by the Center but also by the Department of Mathematics and other mathematics centers at POSTECH at a glance. Also available on the website are the preprints of the members of the Center and the database for the entire collection of the CGP library which can be searched by title, author, ISBN, or year of publication.

In addition, video recordings of most talks, lectures, and conferences hosted by the CGP are uploaded and made available on the website with the consent of speakers. This feature allows anyone who is interested to access and benefit from the talks regardless of their physical location.

For the convenience of visitors and job applicants and the efficiency of the application process, the Center has implemented application features on its website. Those who are interested in visiting and conducting collaborative research with the members of the Center or who are interested in a research position at the Center can apply online at the website.

The CGP Hall, where most of the talks hosted by the Center are given, is equipped with a projector system including a Mac computer dedicated for the purpose of presentations and a Windows computer for general use.

The CGP operates a Linux-based high-performance computation server that can be used to run several CAS (computer algebra systems) such as Mathematica and Maple. The Center also provides web-publishing services for conferences and seminars as well as research-related materials and personal webpages.



## CGP Guesthouse

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The Center aims to facilitate the active creation of new research and the dissemination of recent progress at the boundary of what is known. CGP has created a comprehensive visitor programs to attract researchers from both Korea and abroad.

The Center operates a fully-furnished apartment-style guesthouse to better accommodate our visitors, especially those who are visiting with their family, and/or visiting for an extended period of time. The guesthouse is located near POSTECH campus, approximately 20 minutes walking away from the Center.



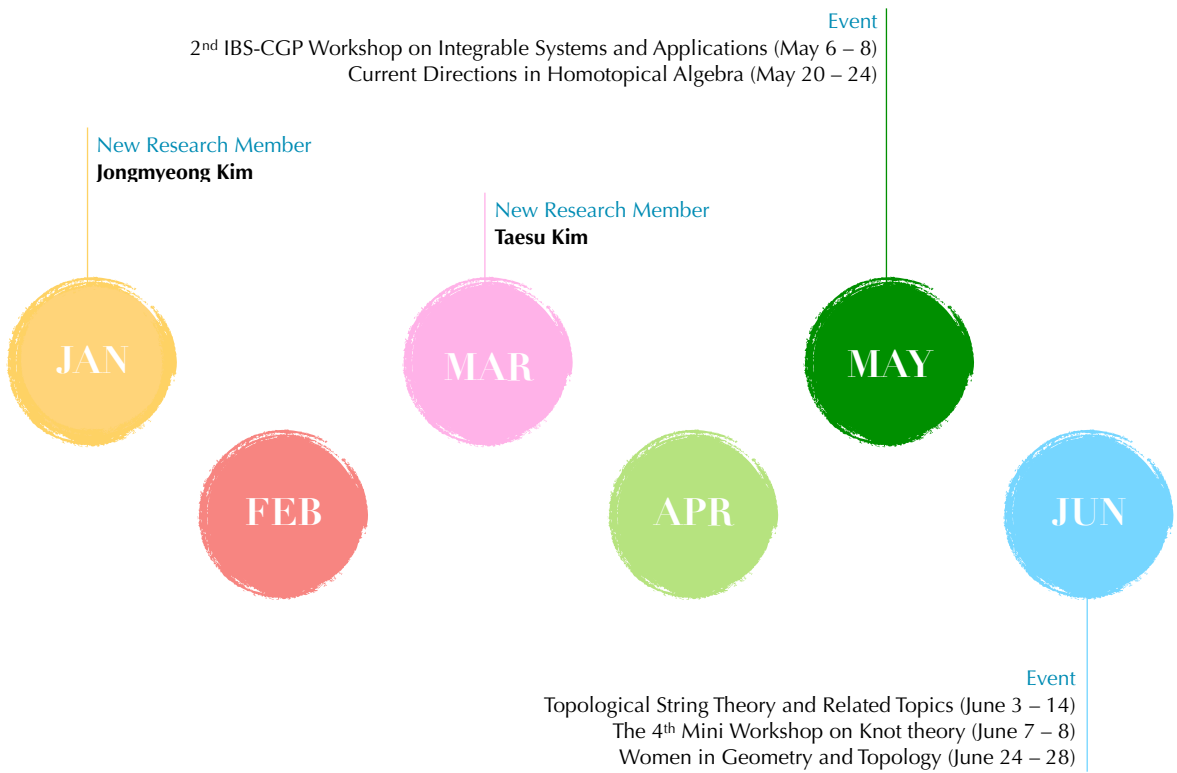
## Scientific Activities



### Statistics (2019 – 2020)

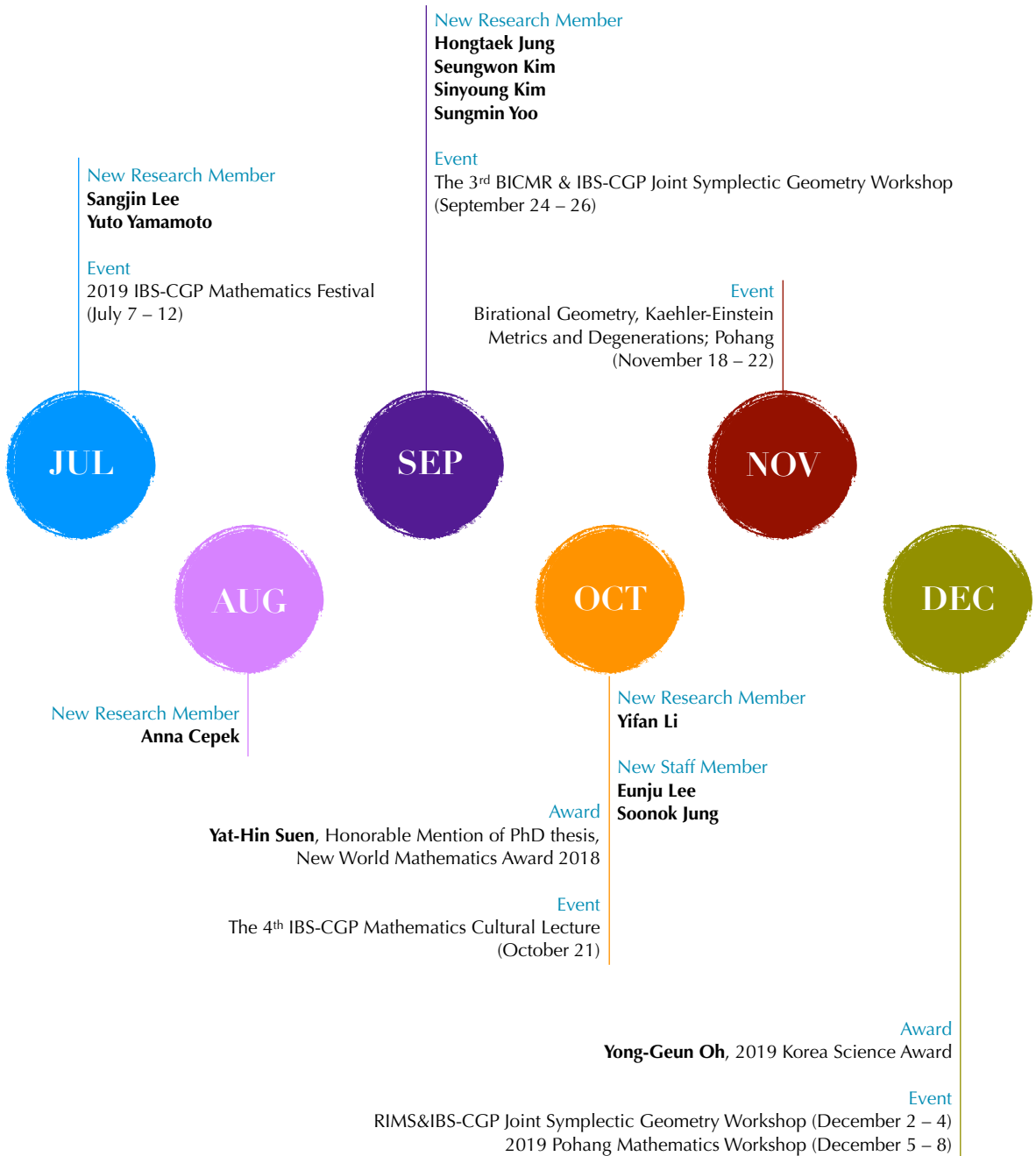
- 9 conferences
- 110 talks and seminars
- 8 lecture series
- 2 public events
- 154 visits by 86 visitors

## CGP at a Glance



2019

2019



# CGP at a Glance



JAN

MAR

MAY

FEB

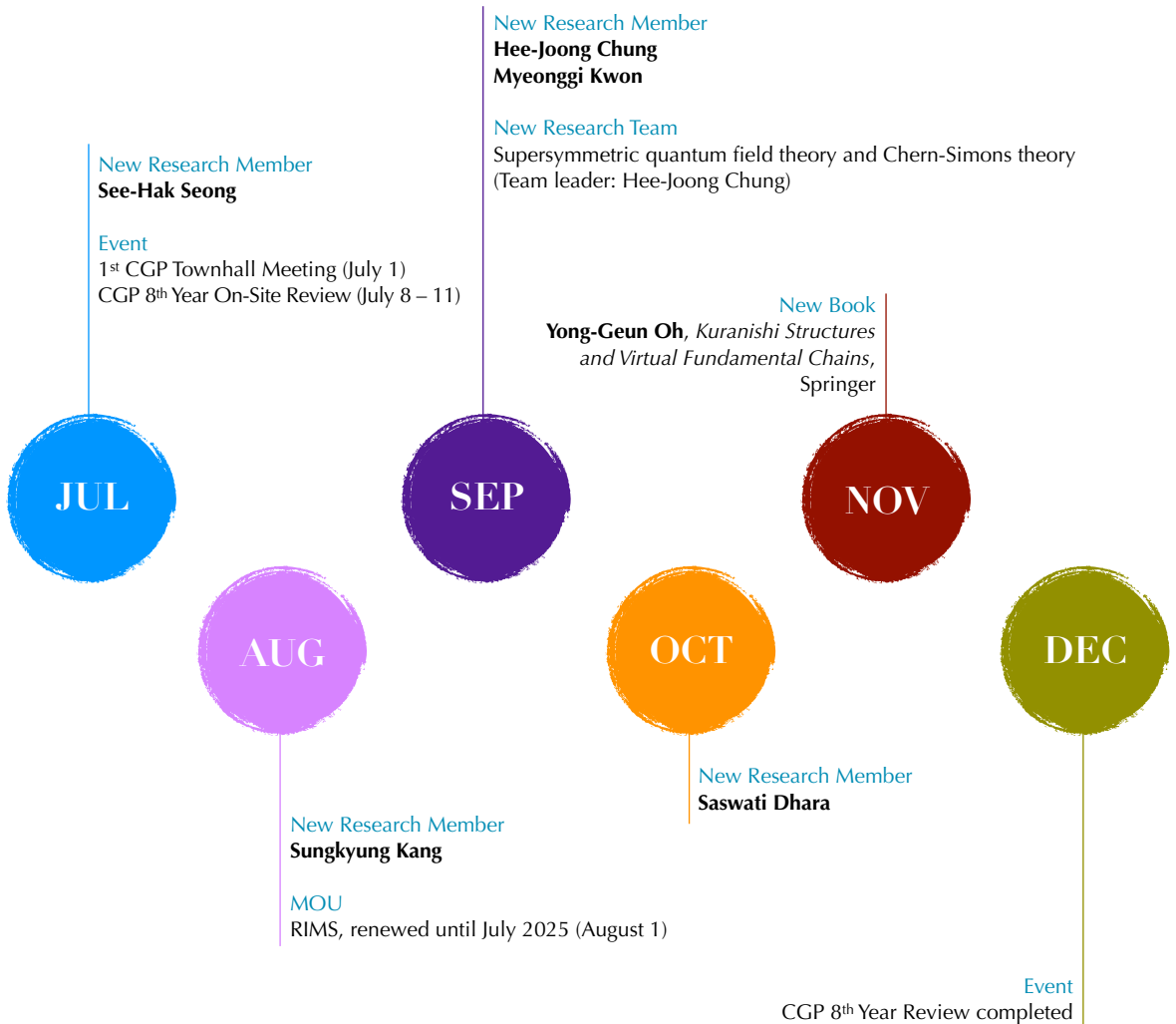
APR

JUN

2020



# 2020



## Conferences

In an effort to take on a leading role in enriching the mathematical society, the Center works in collaboration with other institutes and organizations to hold conferences for a wider audience of mathematicians and scholars. The Center has organized or co-organized 9 conferences:

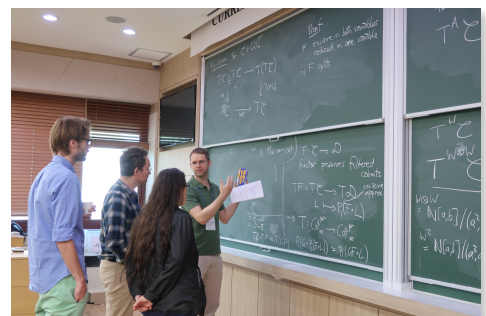
- **2nd IBS-CGP Workshop on Integrable Systems and Applications**; May 6 – 8, 2019

- Organizers: Alexander Aleksandrov (IBS-CGP)
- Invited Speakers: Guido Carlet (University of Bourgogne)  
Johan van de Leur (Utrecht University)  
Oleg Lisovyi (University of Tours)  
Todor Milanov (Kavli IPMU\*)  
Paul Norbury (University of Melbourne)  
Kanehisa Takasaki (Kindai University)  
Andrei Zotov (Steklov Mathematical Institute)



- **Current Directions in Homotopical Algebra**; May 20 – 24, 2019

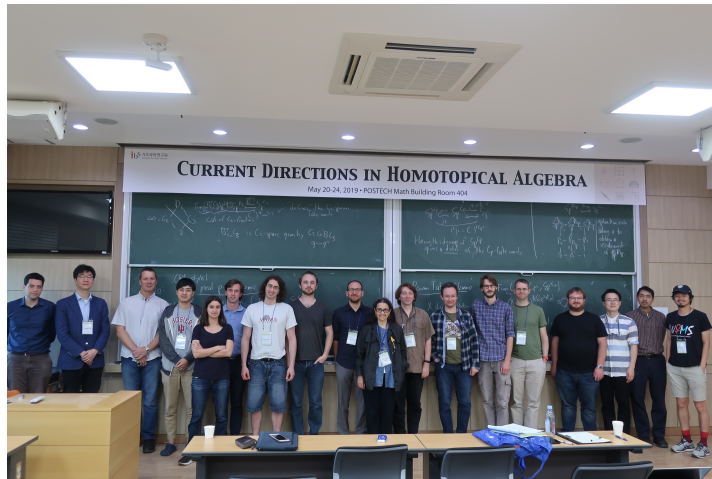
- Organizers: Gabriel C. Drummond-Cole (IBS-CGP); Rune Haugseng (IBS-CGP)
- Invited Speakers: David Carchedi (George Mason)  
Imma Gálvez Carrillo (Universitat Politècnica de Catalunya)  
Michael Ching (Amherst College)  
Gijs Heuts (Utrecht University)  
Inbar Klang (EPFL)<sup>†</sup>  
Joachim Kock (Universitat Autònoma de Barcelona)  
Damien Lejay (IBS-CGP)  
Andrew Macpherson (Kavli IPMU\*)  
Joost Nuiten (Universite de Montpellier)  
Jay Shah (University of Notre Dame)  
Daniel Stevenson (University of Adelaide)  
Hiro Lee Tanaka (MSRI)<sup>‡</sup>  
Andrew Tonks (University of Leicester)



\* Kavli Institute for the Physics and Mathematics of the Universe

<sup>†</sup> École Polytechnique Fédérale de Lausanne

<sup>‡</sup> Mathematical Sciences Research Institute



• **Topological String Theory and Related Topics**; June 3 – 14, 2019

- Organizers: Ahmad Zein Assi (Universität Bern); Alba Grassi (University of Geneva)  
Cheol-Hyun Cho (Seoul National University); Calin Lazaroiu (IBS-CGP)  
Sangmin Lee (Seoul National University); Wolfgang Lerche (CERN\*)  
Marcos Marino (University of Geneva); Nikita Nekrasov (Simons Center for Geometry and Physics);  
Yong-Geun Oh (IBS-CGP); Johannes Walcher (Universität Heidelberg)
- Key speakers: Albrecht Klemm (Physikalisches Institut)  
Kumar Shiv Narain (ICTP†)  
Ricardo Schiappa (Instituto Superior Técnico, University of Lisbon)  
Cheol-Hyun Cho (Seoul National University)  
Jae-Suk Park (POSTECH)  
Jake Solomon (Hebrew University)

• **The 4th Mini Workshop on Knot Theory**; June 7 – 8, 2019

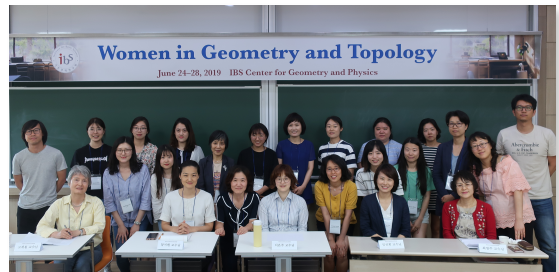
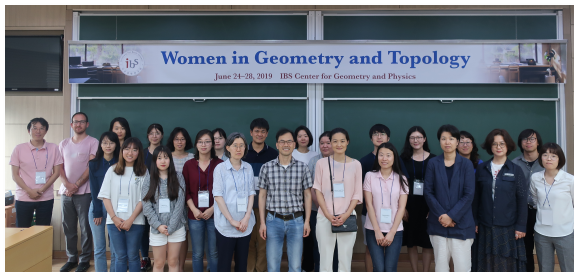
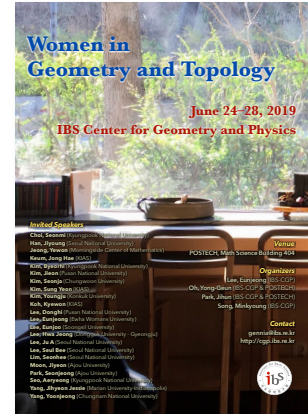
- Organizers: Byunghee An (IBS-CGP); Hwa Jeong Lee (Dongguk University - Gyeongju)
- Invited Speakers: Byunghee An (IBS-CGP)  
Hyoungjun Kim (Ewha Woman University)  
Sangyop Lee (Chung-Ang University)  
Jung Hoon Lee (Chonbuk National University)  
Mikyung Lim (KAIST)  
Sungjong No (Korea University)  
Seungsang Oh (Korea University)  
Alexander Stoimenow (GIST)

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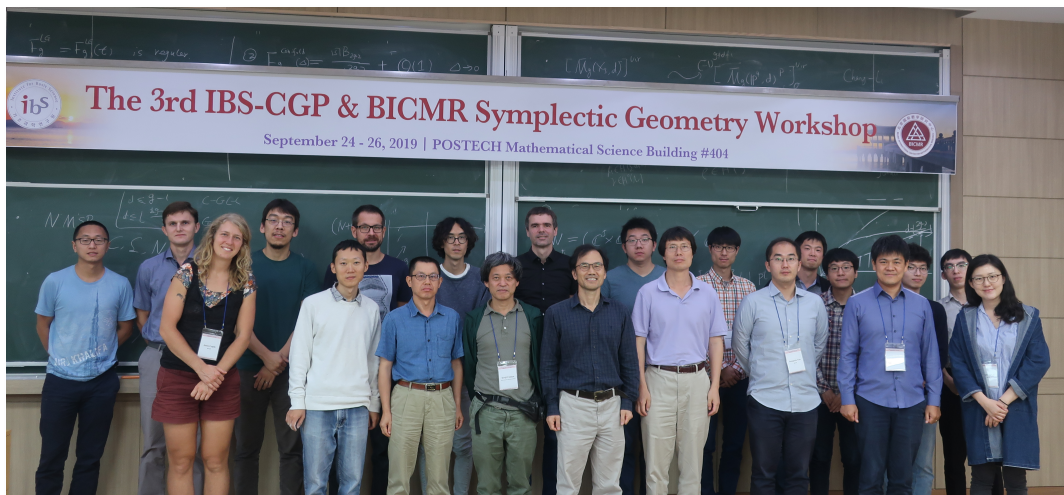
\* Conseil Européenne pour la Recherche Nucléaire

† International Centre for Theoretical Physics

- **Women in Geometry and Topology**; June 24 – 28, 2019
  - Organizers: Eunjeong Lee (IBS-CGP); Yong-Geun Oh (IBS-CGP); Jihun Park (IBS-CGP); Minkyong Song (IBS-CGP)
  - Invited Speakers: Seonmi Choi (Kyungpook National University)
    - Jiyoung Han (Seoul National University)
    - Yewon Jeong (Chinese Academy of Sciences)
    - JongHae Keum (KIAS)
    - Byeorhi Kim (Kyungpook National University)
    - Jieon Kim (Pusan National University)
    - Seonja Kim (Chungwoon University)
    - Sung Yeon Kim (KIAS)
    - Youngju Kim (Konkuk University)
    - Donghi Lee (Pusan National University)
    - Eunjeong Lee (Ehwa Womans University)
    - Eunjoo Lee (Soongsil University)
    - Hwa Jeong Lee (Dongguk University - Gyeongju)
    - Ju A Lee (Seoul National University)
    - Seul Bee Lee (Seoul National University)
    - Seonhee Lim (Seoul National University)
    - Jiyeon Moon (Ajou University)
    - Kyewon Koh Park (KIAS)
    - Seonjeong Park (Ajou University)
    - Aeryeong Seo (Kyungpook National University)
    - Jihyeon Jessie Yang (Marian University-Indianapolis)
    - Yoonjeong Yang (Chungnam National University)



- **The 3rd IBS-CGP & BICMR Joint Symplectic Geometry Workshop**; September 24 – 26, 2019
  - Organizers: Xiaobu Liu (BICMR\*, Peking University); Yong-Geun Oh (IBS-CGP);  
Gang Tian (BICMR\*, Peking University, Princeton University)
  - Invited Speakers: Alexander Aleksandrov (IBS-CGP)
    - Alexandr Buryak (University of Leeds)
    - Huijun Fan (Peking University)
    - Kenji Fukaya (Simons Center for Geometry and Physics)
    - Shuai Guo (Peking University)
    - Jianxun Hu (Sun Yat-Sen University)
    - Hyenho Lho (ETH Zürich†)
    - Emanuel Scheidegger (BICMR\*, Peking University)
    - Longting Wu (ETH Zürich†)
    - Guangbo Xu (Texas A&M University)




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\* Beijing International Center for Mathematical Research

† Eidgenössische Technische Hochschule Zürich

• **Birational Geometry, Kähler-Einstein Metrics and Degenerations; Moscow-Shanghai-Pohang**

April 8 – 13 (Moscow); June 10 – 14 (Shanghai); November 18 – 22, 2019 (Pohang)

- Organizers: Ivan Cheltsov (University of Edinburgh); Xiuxiong Chen (Stony Brook University)

Ludmil Katzarkov (University of Miami); Jihun Park (IBS-CGP)

- Invited Speakers (Pohang): Jacob Cable (University of Manchester)

Paolo Cascini (Imperial College London)

Ivan Cheltsov (University of Edinburgh)

Jiang Chen (Fudan University)

Sung Rak Choi (Yeonsei University)

Thibaut Delcroix (Université de Montpellier)

Alexei Golota (National Research University Higher School of Economics)

Igor Krylov (KIAS)

Takuzo Okada (Saga University)

Jinhyung Park (Sogang University)

Yuri Prokhorov (Steklov Mathematical Institute)

Victor Przyjalkowski (Steklov Mathematical Institute)

Julius Ross (University of Illinois at Chicago)

Yanir Rubinstein (University of Maryland)

Taro Sano (Kobe University)

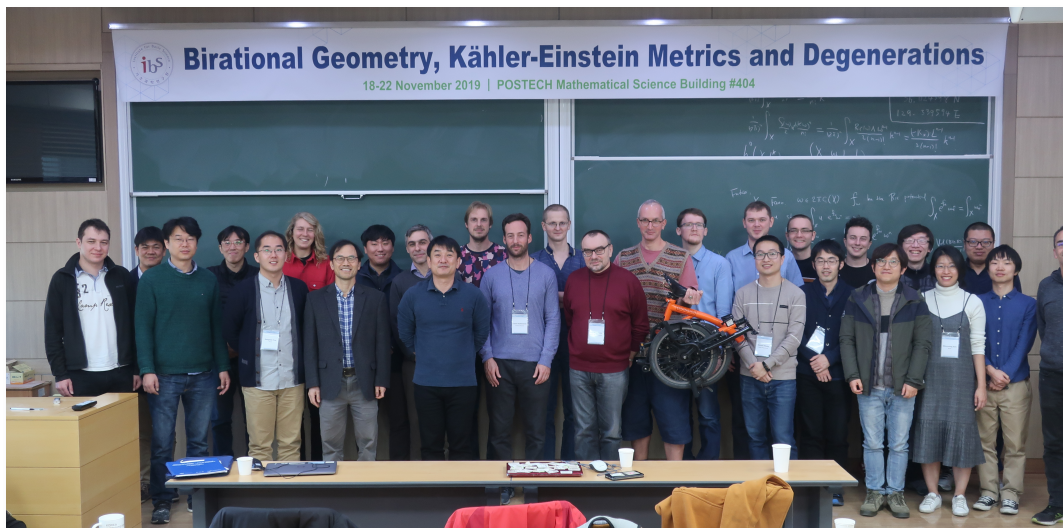
Constantin Shramov (National Research University Higher School of Economics)

Andrey Trepalin (Institute for Information Transmission Problems)

David Witt Nystrom (University of Gothenburg)

Joonyeong Won (KIAS)

Kewei Zhang (BICMR\*)



\* Beijing International Center for Mathematical Research

- **RIMS&IBS-CGP Joint Symplectic Geometry Workshop**; December 2 – 4, 2019
  - Organizers: Yong-Geun Oh (IBS-CGP); Kaoru Ono (RIMS\*)
  - Invited Speakers: Jae-Young Choi (POSTECH)
    - Masahiro Futaki (Chiba University)
    - Jean Gutt (INU Champollion†)
    - Jongmyeong Kim (IBS-CGP)
    - Sangjin Lee (IBS-CGP)
    - Weonmo Lee (POSTECH)
    - Cheuk Yu Mak (University of Cambridge)
    - Zachary Sylvan (Columbia University)
    - Toru Yoshiyasu (Kyoto University)
    - Jun Zhang (Universite de Montreal)
  
- **2019 Pohang Mathematics Workshop**; December 5 – 8, 2019
  - Organizers: Sangjin Lee (IBS-CGP); Jun Yong Park (IBS-CGP)
  - Invited Speakers: Youngjin Bae (KIAS)
    - Clifford Blakestad (POSTECH)
    - Sungmun Cho (POSTECH)
    - Yonghwa Cho (KIAS)
    - Yunhyung Cho (Sungkyunkwan University)
    - Hongtaek Jung (IBS-CGP)
    - Kunwoo Kim (POSTECH)
    - Seungwon Kim (IBS-CGP)
    - Shinyoung Kim (IBS-CGP)
    - Sungmin Yoo (IBS-CGP)



\* Research Institute for Mathematical Sciences

† Institut National Universitaire Champollion

## Seminars

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The CGP hosts various seminars given both by visiting scholars and the members of the Center.

### **Derived Seminar** (Mondays 13:30 – 15:30)

The derived seminar is a weekly working seminar focusing on derived tools such as model categories,  $\infty$ -categories, DG-categories, etc. The goal of this seminar is to become more familiar with these modern mathematical tools.

### **Symplectic Monday** (Mondays 16:00 – 18:00)

The talks are focused on symplectic geometry and chaired by Director Yong-Geun Oh.

### **Algebraic Geometry Seminar** (Tuesdays 16:00 – 18:00)

The talks are focused on algebraic geometry and chaired by Associate Director Jihun Park.

### **Wednesday Noon Seminar** (Wednesdays 12:00 – 13:00)

The Wednesday Noon Seminar runs weekly with lunch for talks by CGP members on various topics of their own research interest or current works. This is kind of semi-closed seminar open to CGP members and visitors only.

### **Director's seminar** (Bi-weekly Wednesdays 13:30 – 15:30)

The purpose of this seminar is to give updates on current developments and mathematical research highlights in general to CGP members and visitors, and to promote deeper interaction between the speaker and the audience. The seminar's general spirit reflects that of the famous Gelfand Seminar.

### **The Center for Geometry and Physics Seminar** (Thursdays 16:00 – 18:00)

The Center for Geometry and Physics Seminar on every Thursday afternoon is the most important regular event of the CGP, and generally all members of the Center participate. The seminars are formatted to encourage robust and dynamic interactions among participants. The seminar is structured as a two-hour talk by a designated speaker with a thirty minute intermission with tea and snack. The first half is intended to be a colloquium-level talk suitable for a general mathematical audience, while the second half can be more specialized. Discussions may continue over dinner.

### **Mathematical Physics Seminar** (Fridays 13:00 – 15:00)

The talks are focused on mathematical physics chaired by Director Yong-Geun Oh and a research fellow, Alexander Aleksandrov.

### **IBS-CGP Post-doc Lecture Series**

The Director encourages CGP post-doc members to give 3-4 one-hour lectures on their research area. The main purpose of the series is to train post-doc's lecture skills so the Director comments on the lecture series and give suggestions to improve.

## List of All Talks

2020

### *Jacobi–Trudi formulas for flagged refined dual stable Grothendieck polynomials*

**Jang Soo Kim** (Sungkyunkwan University)  
November 25, 2020

[IBS-CGP Post-doc Lecture Series]

### *Introduction to topological Fukaya categories I – III*

**Sangjin Lee** (IBS-CGP)  
October 26 – 30, 2020

### *Adjoint Reidemeister torsion and 3D-3D correspondence*

**Dongmin Gang** (Asia Pacific Center for Theoretical Physics)  
October 16, 2020

[IBS-CGP Post-doc Lecture Series]

### *Deformation of complex structures via complete Kähler metrics I – III*

**Sungmin Yoo** (IBS-CGP)  
September 21 – 25, 2020

### *Arithmetic of the moduli of fibrations & Number theory*

**Jun Yong Park** (IBS-CGP)  
August 25, 2020

### *Arithmetic of the moduli of algebraic curves and Abelian varieties over global fields*

**Jun Yong Park** (IBS-CGP)  
August 18, 2020

### *Minimal degree rational curves on moduli space of symplectic and orthogonal bundles on a curve*

**Sanghyeon Lee** (KIAS)  
August 11, 2020

### *Classification of hyperbolic Dehn fillings*

**BoGwang Jeon** (POSTECH)  
July 30, 2020

### *Connectivity and the nef cone of the Hilbert scheme of hypersurfaces in the Grassmannian*

**See-Hak Seong** (IBS-CGP)  
July 14, 2020

[Intensive Lecture]

### *Overview of Springer theory I – V*

**Dongkwan Kim** (University of Minnesota)  
June 8 – 12, 2020

### *Two-row $W$ -graphs in affine type $A$*

**Dongkwan Kim** (University of Minnesota)  
June 11, 2020

### *Cohomological rigidity problems for toric varieties (especially for Bott manifolds)*

**Eunjeong Lee** (IBS-CGP)  
May 26, 2020

### *Birational Calabi-Yau manifolds have the same small quantum product*

**Yifan Li** (IBS-CGP)  
May 20, 2020

### *Bott manifolds and Grossberg-Karshon twisted cubes*

**Eunjeong Lee** (IBS-CGP)  
May 19, 2020

### *Birational Calabi-Yau $n$ -folds have the same Zeta functions*

**Jun Yong Park** (IBS-CGP)  
May 13, 2020

### *Quantum cohomology ring of birational Calabi-Yau manifolds*

**Yong-Geun Oh** (IBS-CGP, POSTECH)  
May 6, 2020

### *Symplectic cohomology of Liouville sectors II*

**Dogancan Karabas** (Northwestern University)  
February 18, 2020



***M. Kontsevich's graph complexes and universal structures on graded symplectic manifolds***

**Kevin Morand** (Sogang University)  
February 7, 2020

***Cluster structure for Vianna's Lagrangian tori***

**Yong-Geun Oh** (IBS-CGP, POSTECH)  
February 5, 2020

***Fiberwise Kähler-Einstein metric and Kähler-Ricci flow on a family of strongly pseudoconvex domains***

**Young-Jun Choi** (Pusan National University)  
January 30, 2020

***Cluster algebras, quiver representations and Caldero-Chapoton formula***

**Jongmyeong Kim** (IBS-CGP)  
January 22, 2020

***Weakly 1-completeness of fiber bundles over compact Kähler manifolds I – II***

**Aeryeong Seo** (Kyungpook National University)  
January 20 – 21, 2020

***Virtual intersection theories***

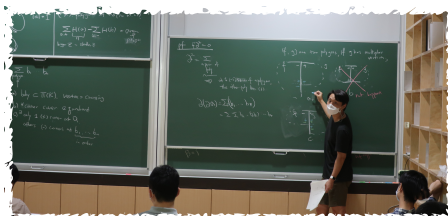
**Hyeonjun Park** (Seoul National University)  
January 16, 2020

***Mapping spaces I***

**Anna Cepek** (IBS-CGP)  
January 13, 2020

***Birational geometry of moduli spaces of curves and  $K3$  surfaces***

**Changho Han** (University of Georgia)  
January 2, 2020



2019

***Link homology theories, ribbon concordances, and their generalizations***

**Sungkyung Kang** (The Chinese University of Hong Kong)  
December 19, 2019

***Introduction to cluster structures II***

**Eunjeong Lee** (IBS-CGP)  
December 18, 2019

***On the way to a proof of the dlt extension of the abundance conjecture***

**Tsz On Mario Chan** (Pusan National University)  
December 17, 2019

***Artin's primitive root conjecture for function fields without Riemann hypothesis***

**Seoyoung Kim** (Queen's University)  
December 12, 2019

***Introduction to cluster structures I***

**Eunjeong Lee** (IBS-CGP)  
December 11, 2019

***Compactifications of rational curve spaces in del Pezzo  $\mathbb{P}^3$ -fold***

**Kiryong Chung** (Kyungpook National University)  
December 10, 2019

***Introduction to configuration spaces II***

**Byung Hee An** (IBS-CGP)  
December 2, 2019

***Doubling stops and spherical swaps***

**Zachary Sylvan** (Columbia University)  
November 28, 2019

***Organizational meeting for 'Cluster algebra and related topics'***

**Yong-Geun Oh** (IBS-CGP, POSTECH)  
November 27, 2019

***A survey for LG/LG mirror symmetry***

**Yifan Li** (IBS-CGP)  
November 25, 2019

***Introduction to configuration spaces I***

**Anna Cepek** (IBS-CGP)  
November 25, 2019

***Characteristic numbers of elliptic fourfolds***

**Jinwoo Kang** (California Institute of Technology)  
November 14, 2019

***Wall-crossing formula and Floer theory VI***

**Yong-Geun Oh** (IBS-CGP, POSTECH)  
November 13, 2019

**Wall-crossing formula and Floer theory V****Yong-Geun Oh** (IBS-CGP, POSTECH)

November 6, 2019

**Categorical systolic inequality for the Fukaya category of 4-dimensional Milnor fiber of ADE singularity****Jongmyeong Kim** (IBS-CGP)

October 31, 2019

**Wall-crossing formula and Floer theory IV****Yong-Geun Oh** (IBS-CGP, POSTECH)

October 23, 2019

[IBS-CGP Post-doc Lecture Series]

**Configuration spaces of  $\mathbb{S}^1$  and  $\mathbb{R}^n$  by way of higher categories I – III****Anna Ceppek** (IBS-CGP)

October 14 – 17, 2019

**Towards A+B theory in conifold transitions for Calabi-Yau threefolds****Yuan-Pin Lee** (University of Utah)

October 10, 2019

**Tropical geometry and Hodge theory****Yuto Yamamoto** (IBS-CGP)

October 1, 2019

**Wall-crossing formula and Floer theory III****Yong-Geun Oh** (IBS-CGP, POSTECH)

September 18, 2019

**On SYZ fibrations****Yat-Hin Suen** (IBS-CGP)

September 4, 2019

[Intensive Lecture]

**Cluster algebras and Newton-Okounkov bodies I – III****Naoki Fujita** (The University of Tokyo)

August 26 – 28, 2019

**On a GIT characterization of cofree representations****Matthew Satriano** (University of Waterloo)

August 22, 2019

**Wall crossing formula and Floer homology II****Yong-Geun Oh** (IBS-CGP, POSTECH)

August 21, 2019

**A higher-dimensional generalization of pseudo-Anosov surface automorphisms****Sangjin Lee** (IBS-CGP)

August 19, 2019

**Wall crossing formula and Floer homology I****Yong-Geun Oh** (IBS-CGP, POSTECH)

August 14, 2019

**Quotient singularities of algebraic surfaces with small Betti numbers****Kyoung-Seog Lee** (IBS-CGP)

August 13, 2019

**BPS invariants for Seifert manifolds****Hee-Joong Chung** (Tsinghua University)

August 8, 2019

**Towards homological mirror symmetry for cluster quivers****Kyungyong Lee** (University of Alabama)

August 5, 2019

**Exact Lagrangian fillings of Legendrian torus knots****Youngjin Bae** (RIMS\*)

August 2, 2019

**Double Poisson algebras up to homotopy****Johan Leray** (University of Paris 13)

August 1, 2019

**Augmentations and sheaves for Legendrian graphs****Tao Su** (École Normale Supérieure - CNRS)

July 31, 2019

**Cox rings and combinatorics****Kyoung-Seog Lee** (IBS-CGP)

July 23, 2019

**Del Pezzo surfaces: toric systems, degenerations, and minimal model program****Yonghwa Cho** (KIAS)

July 16, 2019

**Motivic integrations****Kyoung-Seog Lee** (IBS-CGP)

July 15, 2019

**On span of generalized Dold manifolds****Soumen Sarkar** (Indian Institute of Technology Madras)

July 9, 2019

**Tropical geometry and Newton-Okounkov cones for Grassmannian of planes from compactifications****Jihyeon Jessie Yang** (Marian University-Indianapolis)

July 8, 2019

**Introduction to Newton-Okounkov theory****Jihyeon Jessie Yang** (Marian University-Indianapolis)

July 8, 2019

**Degenerations in algebra, geometry, and combinatorics from the tropical viewpoint****Jihyeon Jessie Yang** (Marian University-Indianapolis)

July 8, 2019

\* Research Institute for Mathematics Sciences

***Chow cohomology rings of complete flag varieties and Gelfand-Zetlin toric varieties*****Kyeong-Dong Park** (IBS-CGP)

July 2, 2019

***Entropy of symplectomorphisms via Lagrangian cobordisms*****Jongmyeong Kim** (IBS-CGP)

June 17, 2019

***Involutive Lie bialgebras: Algebra II*****Gabriel C. Drummond-Cole** (IBS-CGP)

June 5, 2019

***Localizing quantum Chern classes*****Yasha Savelyev** (University of Colima)

May 30, 2019

***Variation of Kähler-Einstein metrics on noncompact fibrations*****Sungmin Yoo** (KIAS)

May 27, 2019

***Quantum toroidal algebras and instantons in supersymmetric gauge theories*****Jean-Emile Bourgin** (KIAS)

May 24, 2019

***Involutive Lie bialgebras: Algebra I*****Gabriel C. Drummond-Cole** (IBS-CGP)

May 15, 2019

 ***$\mathbb{S}^3$ -folds of general type and canonically of fiber type with high genus*****YongJoo Shin** (KIAS)

May 14, 2019

***Index theorems for gauge theories, wall crossing and holonomy saddles*****Piljin Yi** (KIAS)

April 26, 2019

***Introduction to string polytopes and their combinatorics*****Eunjeong Lee** (IBS-CGP)

April 24, 2019

***Chow and Voevodsky motives of moduli spaces of vector bundles on curves*****Kyoung-Seog Lee** (IBS-CGP)

April 23, 2019

***Introduction to string polytopes and their combinatorics*****Eunjeong Lee** (IBS-CGP)

April 22, 2019

***From walking backgrounds to the early universe*****Lilia Anguelova** (Institute for Nuclear Research and Nuclear Energy)

April 19, 2019

***Sheaf cohomology*****Gabriel C. Drummond-Cole** (IBS-CGP)

April 8, 2019

***Fundamental Factorization of a GLSM*****Bumsig Kim** (KIAS)

April 5, 2019

***Sheaves and sites*****Gabriel C. Drummond-Cole** (IBS-CGP)

April 1, 2019

***Langlands duality and quantum field theory*****Philsang Yoo** (Yale University)

March 27, 2019

***Bundles on prime Fano threefolds of degree 22*****Kyeong-Dong Park** (IBS-CGP)

March 26, 2019

***Introduction to Voevodsky motives*****Kyoung-Seog Lee** (IBS-CGP)

March 25, 2019

***Brauer groups and rational points on varieties*****Taekyung Kim** (IBS-CGP)

March 19, 2019

***Introduction to Chow motives*****Kyoung-Seog Lee** (IBS-CGP)

March 18, 2019

***Uncertainty principle, Hamiltonian dynamics and Gromov's nonsqueezing theorem*****Yong-Geun Oh** (IBS-CGP, POSTECH)

March 15, 2019

***Fukaya category, hyperbolic geometry and knot invariants*****Yong-Geun Oh** (IBS-CGP, POSTECH)

March 15, 2019

***Characterizing symplectic Grassmannians by varieties of minimal rational tangents*****Qifeng Li** (KIAS)

March 14, 2019

***Fano deformation rigidity of rational homogeneous spaces*****Qifeng Li** (KIAS)

March 12, 2019

***2d TQFTs from Calabi-Yau varieties*****Jongmyeong Kim** (IBS-CGP)

March 11, 2019

[Intensive Lecture]

*Topology of Hessenberg varieties and related topics I – III*

**Mikiya Masuda** (Osaka City University)

March 6 – 8, 2019

[Intensive Lecture]

*An introduction to algebraic K-theory I – IV*

**Bhamidi Sreedhar** (KIAS)

March 5 – 8, 2019

*On the motivic sphere spectrum and Hilbert schemes*

**Elden Elmanto** (MSRI\*)

March 5, 2019

*Introduction to defect 2d topological field theory*

**Yat-Hin Suen** (IBS-CGP)

March 4, 2019

*Mapping coalgebras*

**Brice Le Grignou** (University of Utrecht)

February 28, 2019

*Product formula for volumes*

**Sung Rak Choi** (Yonsei University)

February 26, 2019

*Yet another introduction to algebraic K-theory*

**Rune Haugseng** (IBS-CGP)

February 26, 2019

*Quasimaps to relative GIT quotients and applications*

**Jeongseok Oh** (KIAS)

February 25, 2019

*Polynomial invariants of spatial graphs*

**Youngsik Huh** (Hanyang University)

February 25, 2019

*Algebraic K-theory and motivic cohomology I – III*

**Jinhyun Park** (KAIST)

February 21 – 22, 2019

*Geometry and topology of surfaces isogenous to a product and their relatives*

**Kyoung-Seog Lee** (IBS-CGP)

February 19, 2019

*T-structures II*

**Damien Lejay** (IBS-CGP)

February 19, 2019

*Arboreal singularities*

**Byung Hee An** (IBS-CGP)

February 13, 2019

*Delta-invariants of complete intersection log del Pezzo surfaces*

**In-kyun Kim** (Seoul National University)

February 12, 2019

*t-structures on triangulated categories*

**Taesu Kim**

February 12, 2019

*A connected sum conjecture and hyperbolic knots*

**Seonhwa Kim** (IBS-CGP)

February 11, 2019

*BPS/CFT correspondence: some applications of defects in supersymmetric gauge theory*

**Nikita Nekrasov** (Simons Center for Geometry and Physics)

February 8, 2019

*Reminder on triangulated categories*

**Yong-Geun Oh** (IBS-CGP, POSTECH)

January 29, 2019

[IBS-CGP Post-doc Lecture Series]

*Lectures on SYZ I – III*

**Yat-Hin Suen** (IBS-CGP)

January 28 – 30, 2019

*On Hirschowitz's conjecture on the formal principle*

**Jun-Muk Hwang** (KIAS)

January 18, 2019

*Introduction to spectra*

**Damien Lejay** (IBS-CGP)

January 15, 2019

*Lie algebra representations and moduli spaces of sheaves on surfaces*

**Sheldon Katz** (University of Illinois)

January 11, 2019

*Notion of motives and Grothendieck's proposal*

**Jun Yong Park** (IBS-CGP)

January 8, 2019

*LaTeX lectures for mathematicians*

**Hyunwoo Kwon** (Sogang University)

January 4, 2019

*Elliptic equations with singular drifts on Lipschitz domains*

**Hyunwoo Kwon** (Sogang University)

January 4, 2019

\* Mathematical Sciences Research Institute

## Visitor Programs and Visitors

CGP runs programs to support visiting scholars. The goal of the visitor programs at the center is to support dynamic researchers working in topics related to the core fields of interest. In particular, the center aims to facilitate the active creation of new research and the dissemination of recent progress at the boundary of what is known. More concretely, we believe that mixing interesting people working on interesting problems in one place has the potential to reveal commonalities, promote collaboration, and help those people advance in understanding.

The center can provide office space and housing for approved visitors. Limited funds are available to support for travel and local expenses for visiting scholars.



## List of All Visitors

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2020

**JongHae Keum** (KIAS)

October 27 – November 12, 2020

**Kyeong-Dong Park** (KIAS)

October 19 – 23, 2020

**Dongmin Gang** (Asia Pacific Center for Theoretical Physics)

October 16, 2020

**Minhoon Kim** (Chonnam National University)

September 28 – 29, 2020

September 24 – 26, 2020

**Joonyeong Won** (KIAS)

September 24 – 26, 2020

August 18 – 21, 2020

June 22 – 25, 2020

**Sanghyeon Lee** (KIAS)

August 10 – 15, 2020

**Byung Hee An** (Kyungpook National University)

October 16 – 18, 2020

August 6 – 8, 2020

July 8, 2020

June 12, 2020

**Youngjin Bae** (KIAS)

August 6 – 8, 2020

July 9 – 11, 2020

**Youngjin Cho** (Kyungpook National University)

August 6 – 7, 2020

**BoGwang Jeon** (POSTECH)

July 30, 2020

**Seonjeong Park** (KAIST)

July 28 – 31, 2020

January 9 – 11, 2020

**Yunhyung Cho** (Sungkyunkwan University)

July 28 – 30, 2020

**Jaehyun Hong** (KIAS)

July 20 – 24, 2020

February 10 – 14, 2020

**Soojin Cho** (Ajou University)

July 20 – 24, 2020

February 10 – 14, 2020

**Jihun Yum** (Pusan National University)

July 19 – 24, 2020

**Young-Jun Choi** (Pusan National University)

July 7 – 10, 2020

January 30 – 31, 2020

**Dongkwan Kim** (University of Minnesota)

June 8 – 14, 2020

**Dogancan Karabas** (Northwestern University)

February 18, 2020

**Kevin Morand** (Sogang University)

February 6 – 8, 2020

**Aeryeong Seo** (Kyungpook National University)

January 20 – 22, 2020

**Hyeonjun Park** (Seoul National University)

January 15 – 17, 2020

**Cheol-Hyun Cho** (Seoul National University)

January 14 – 15, 2020

**Seonhwa Kim** (KIAS)

January 6 – 7, 2020 and others

**Changho Han** (University of Georgia)

January 2 – 3, 2020

## 2019

**Tsz On Mario Chan** (Pusan National University)  
December 17 – 18, 2019

**Sungkyung Kang** (The Chinese University of Hong Kong)  
December 15 – 21, 2019

**Seoyoung Kim** (Queen's University)  
December 11 – 13, 2019

**Kiryong Chung** (Kyungpook National University)  
December 10 – 11, 2019

**Zachary Sylvan** (Columbia University)  
November 25 – December 1, 2019

**Yuji Odaka** (Kyoto University)  
November 17 – 20, 2019

**Jinwoo Kang** (California Institute of Technology)  
November 11 – 16, 2019

**Jae Choon Cha** (POSTECH)  
October 18 – 20, 2019

**Yuan-Pin Lee** (University of Utah)  
October 6 – 13, 2019

**Emanuel Scheidegger** (BICMR\*, Peking University)  
September 22 – October 5, 2019

**Joonyeong Won** (KIAS)  
September 16 – 23, 2019

**Dong Youp Suh** (KAIST)  
August 26 – 28, 2019

**Yoosik Kim** (Brandeis University)  
August 26 – 29, 2019

**Naoki Fujita** (The University of Tokyo)  
August 21 – 31, 2019

**Matthew Satriano** (University of Waterloo)  
August 18 – 25, 2019

**Hee-Joong Chung** (Tsinghua University)  
August 5 – 9, 2019

**Kyungyong Lee** (University of Alabama)  
August 4 – 5, 2019

**Johan Leray** (Université Sorbonne Paris Nord)  
July 29 – August 4, 2019

**Tao Su** (École Normale Supérieure - CNRS)  
July 28 – August 3, 2019

**Cheol-Hyun Cho** (Seoul National University)  
July 22 – 23, 2019

**Hansol Hong** (Yonsei University)  
July 22, 2019

**Youngjin Bae** (RIMS<sup>†</sup>)  
July 20 – August 5, 2019

**Yonghwa Cho** (KIAS)  
July 15 – 19, 2019

**Ben Knudsen** (Harvard University)  
July 8 – 19, 2019

**Jihyeon Jessie Yang** (Marian University Indianapolis)  
July 8, 2019

**Jongbaek Song** (KAIST)  
July 8 – 11, 2019

**Soumen Sarkar** (Indian Institute of Technology Madras)  
July 8 – 11, 2019

**Yunhyung Cho** (Sungkyunkwan University)  
July 2 – 4, 2019

**Yoonjeong Yang** (Chungnam National University)  
June 24 – 28, 2019

**Sungmin Yoo** (KIAS)  
May 26 – 28, 2019

**Yasha Savelyev** (University of Colima)  
May 26 – June 2, 2019

**Jean-Emile Bourgine** (KIAS)  
May 23 – 25, 2019

**Joachim Kock** (Universitat Autònoma de Barcelona)  
May 19 – 31, 2019

**Hiro Lee Tanaka** (MSRI<sup>‡</sup>)  
May 14 – 22, 2019

**Yongjoo Shin** (KIAS)  
May 13 – 16, 2019

\* Beijing International Center for Mathematical Research

† Research Institute for Mathematical Sciences

‡ Mathematical Sciences Research Institute

**Piljin Yi** (KIAS)

April 26 – 27, 2019

**Hongyi Chu** (MPIM\*)

April 18 – 26, 2019

**Lilia Anguelova** (Institute for Nuclear Research and Nuclear Energy)

April 14 – 27, 2019

**Bumsig Kim** (KIAS)

April 4 – 6, 2019

**Mirela Elena Babalic** (Horia Hulubei National Institute for Physics and Nuclear Engineering)

April 1 – 21, 2019

**Sung Rak Choi** (Yonsei University)

April 1 – August 31, 2019

February 11 – 28, 2019

**Philsang Yoo** (Yale University)

March 25 – 29, 2019

**Qifeng Li** (KIAS)

March 11 – 14, 2019

**Seonhwa Kim** (KIAS)

March 11, 2019 and others

**Mikiya Masuda** (Osaka City University)

March 5 – 9, 2019

**Tak Kwong Wong** (The University of Hong Kong)

March 5 – 10, 2019

**Bhamidi Sreedhar** (KIAS)

March 4 – 9, 2019

**Elden Elmanto** (MSRI†)

March 1 – 6, 2019

**Hwa Jeong Lee** (DGIST)

February 25 – 27, 2019

**Youngsik Huh** (Hanyang University)

February 25 – 27, 2019

**Brice Le Grignou** (University of Utrecht)

February 24 – March 3, 2019

**Jeongseok Oh** (KIAS)

February 24 – 25, 2019

**Jinhyun Park** (KAIST)

February 20 – 23, 2019

**Seonjeong Park** (Ajou University)

February 14 – 15, 2019

**In-kyun Kim** (Seoul National University)

February 11 – 15, 2019

**Yunhi Cho** (University of Seoul)

February 11 – 15, 2019

January 15 – 16, 2019

**Nikita Nekrasov** (Simons Center for Geometry and Physics)

February 7 – 9, 2019

**Jun-Muk Hwang** (KIAS)

January 18 – 19, 2019

**Keunyoung Jeong** (UNIST)

January 16 – 18, 2019

**Sheldon Katz** (University of Illinois)

January 10 – 13, 2019

**Hyunwoo Kwon** (Sogang University)

January 4, 2019

**Yonghwa Cho** (University of Bayreuth)

January 2 – 4, 2019

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\* Max Planck Institute for Mathematics

† Mathematical Sciences Research Institute

# Public Outreach

## Public Lectures

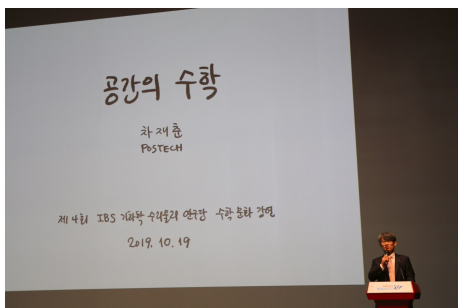
Since 2012 the Center has biannually organized public events with the collaboration of the City of Pohang where the Center is located. The events were popular and well attended by numerous middle and high school students as well as adults.

### 2019

**The 4th IBS-CGP Public Lecture in Mathematics**, Pohang City Hall (Pohang)

“Mathematics of space”, **Jae Choon Cha** (POSTECH), October 19

“Optimal path finding”, **Sang-Il Oum** (IBS, KAIST), October 19



**2017**

**The 3rd IBS-CGP Public Lecture in Mathematics**, POSCO International Center (Pohang)

“Is mathematics a science?”, **Hyungju Park** (Ajou University), October 21

“Climate change from the view point of mathematics”, **Seung-Ki Min** (POSTECH), October 21

**2014**

**The 2nd IBS-CGP Public Lecture in Mathematics**, Pohang City Hall (Pohang)

[Mathematics, the language of nature]

“Mathematics through nature”, **Seung-Yeal Ha** (Seoul National University), October 18

“When a theoretical physicist meets mathematics”, **Seunghwan Kim** (POSTECH), October 18

**2013**

**The Asian Mathematical Conference 2013**, BEXCO (Busan)

“Harmony in mathematics”, **Hong-Jong Kim** (Seoul National University), June 30

“Mathematics of planet earth”, **Christiane Rousseau** (University of Montreal), July 1

**2012**

**The 1st IBS-CGP Public Lecture in Mathematics**, Pohang City Hall (Pohang)

“A look at modern civilization through mathematics”, **Minhyong Kim** (POSTECH, University of Oxford), November 3

In addition to the public lectures organized by the Center, Director **Yong-Geun Oh** and Associate Director **Jihun Park** were invited to speak at the following public outreach efforts:

Seoul Metropolitan Office of Education (Seoul)

“Why math?”, **Yong-Geun Oh**, August 1, 2019

KAOS foundation (Seoul)

“From Fano to Birkar”, **Jihun Park**, September 6, 2018

**Spreading the Wisdom**, National Assembly Library (Seoul)

“Beyond the horizon”, **Yong-Geun Oh**, April 10, 2014

**International Symposium on New Frontiers in Scientific Innovation**, Korea Foundation for Advanced Studies (Seoul)

“Uncertainty principle, symplectic geometry and mirror symmetry”, **Yong-Geun Oh**, July 8, 2014

**Science in Library**, Ulsan Jungbu Library (Ulsan)

“Mathematics - the language of nature”, **Yong-Geun Oh**, September 13, 2014

## Mathematics Festival

One of the prime objectives of the CGP, as a campus research center, is to become a fertile ground for the new generation of mathematicians. In Korea, there have been many well-organized and successful programs aiming for advanced graduate students. On the other hand, similar programs aiming for intermediate and advanced undergraduate students, who wish to be a mathematician, is virtually nonexistent. The Center has planned to begin a week-long annual event titled "IBS-CGP Mathematics Festival" as an attempt to fill the gap.

Each year, the theme of the festival is selected with a focus on mathematicians or mathematical achievements of historic significance. The Center invites up to 20 brilliant undergraduate students from universities in Korea based on motivation and recommendations. Each year, depending on the theme, three to four colloquium lecturers/senior mentors are invited. Additionally two to four postdoctoral researchers are also invited as assistants/junior mentors.

The purpose of the event is to introduce the research of the mentor professors and to help students better understand what it is like to live and work as a mathematician with an ultimate goal of creating a fertile ground for the new generation of mathematics in Korea.

### 2019

**The 5th IBS-CGP Mathematics Festival**, POSTECH (Pohang), July 7 – 12

"Introduction to invariant theory", **Jinhyung Park** (Sogang University)

"Low dimensional topology and geometric group theory", **Hyungryul Baik** (KAIST)

"Fermat's last theorem and modular curves", **Hwajong Yoo** (Seoul National University)

"Introduction to mathematical theory for many particle system", **Donghyun Lee** (POSTECH)

*18 students, 5 professors/senior mentors participated*



**2018**

**The 4th IBS-CGP Mathematics Festival**, POSTECH (Pohang), August 13 – 17

“Representations of symmetric groups and symmetric functions”, **Jae-Hoon Kwon** (Seoul National University)

“An introduction to ergodic Ramsey theory”, **Younghwan Son** (POSTECH)

“Variations on a theme”, **Sung-Jin Oh** (KIAS)

*20 students, 5 professors/senior mentors participated*

**2017**

**The 3rd IBS-CGP Mathematics Festival**, POSTECH (Pohang), August 21 – 25

“Arithmetic of number fields”, **Jeehoon Park** (POSTECH)

“Fourier analysis on  $SU(2)$ ”, **Hun Hee Lee** (Seoul National University)

“Elliptic curves and congruent numbers”, **Bo-Hae Im** (KAIST)

*16 students, 4 professors/senior mentors participated*

**2016**

**The 2nd IBS-CGP Mathematics Festival**, POSTECH (Pohang), July 18 – 22

“Discrete harmonic analysis”, **Joonil Kim** (Yonsei University)

“Discontinuous solutions to nonlinear PDEs”, **Myoungjean Bae** (POSTECH)

“Random walks and spectral gaps on graphs”, **Seon Hee Lim** (Seoul National University)

“Some remarks on arithmetic and geometry”, **Minhyong Kim** (University of Oxford)

*20 students, 10 professors/senior mentors, 4 junior mentors participated*

**2015**

**The 1st IBS-CGP Mathematics Festival**, POSTECH (Pohang), August 17 – 21

“Circle packing - graphs, geometry, groups from Gauss to Gromov”, **Sang-hyun Kim** (Seoul National University)

“Along came determinant”, **Jaehyuk Lee** (Ewha Womans University)

“Algebraic structures of plane curves and matrix factorizations”, **Cheol-Hyun Cho** (Seoul National University)

*18 students, 5 professors/senior mentors, 6 junior mentors participated*



## Awards

Director **Yong-Geun Oh** has won the **2019 Korea Science Award**, recognized for his achievements in mirror symmetry, symplectic topology and establishment of the Lagrangian Floer theory. This award is presented by the **Ministry of Science and ICT and the National Research Foundation of Korea** to promote basic science research in Korea and encourage creative and innovative R&D activities. It recognizes scientists who produced outstanding research achievements by identifying the fundamental principles of natural science.



Dr. **Yat-Hin Suen** was awarded **Honorable Mention of PhD thesis for New World Mathematics Award 2018**. “New World Mathematics Awards” was first initiated and established by Dr. Henry Cheng and Professor Yau Shing Tung in September 2007, with funding from the New World Group and China Young Leaders Development Project. Organized by **ICCM (International Congress of Chinese Mathematicians)** and with the first Awards co-organized by Zhejiang University, “New World Mathematics Awards” encourages outstanding mathematics students of Chinese descendant in their pursuit of mathematical truth and strengthen academic exchanges.

In September 2018, the **Norwegian Mathematical Society** celebrated its 100th anniversary at a national meeting of mathematicians held in Bergen. At the meeting, the society announced a new national prize, **the Viggo Brun Prize**. The prize is named after the noted number theorist Viggo Brun who became an honorary member of the Norwegian Mathematical Society in 1974. The 2018 laureate is Dr. **Rune Haugseng**, who receives the prize for his fundamental contributions to the theory of higher categories.

Associate Director **Jihun Park** has won the **Korean Mathematical Society’s 2017 Best Paper Award** for his paper, “Birational rigid Fano threefold hypersurfaces” published in *Memoirs of the American Mathematical Society*.

On November 29, 2013, Director **Yong-Geun Oh** was elected as a member of the **Korean Academy of Science and Technology**, Korea’s highest integrated think-tank for science and technology.

The **Fellows of the American Mathematical Society** program recognizes members who have made outstanding contributions to the creation, exposition, advancement, communication, and utilization of mathematics. Director **Yong-Geun Oh** was selected as one of the inaugural class fellows of the society on November 1, 2012.

The **Kyung-Ahm Education & Culture Foundation** recognized Director **Yong-Geun Oh**’s seminal contributions in Floer homology and Hamiltonian mechanics with the **Kyung-Ahm Prize** on September 24, 2012. The Kyung-Ahm Prize is one of the highest honors in art, science, and technology in Korea.

## MOUs

The CGP has signed several MOUs for active research collaborations and academic exchanges with the mathematics community.

### Beijing International Center for Mathematical Research (BICMR), China

November 2015 – November 2021 (renewed 2018)

- The 3rd BICMR&IBS-CGP Joint Symplectic Geometry Workshop (September 24 – 26, 2019 @ POSTECH)
- Silk Road Geometry Conference 2018 (June 4 – 8, 2018 @ Gökova Geometry/Topology Institute)
- The 2nd BICMR&IBS-CGP Joint Symplectic Geometry Workshop (September 18 – 22, 2017 @ BICMR)
- BICMR&IBS-CGP Joint Symplectic Geometry Workshop (October 31 – November 4, 2016 @ KAL Hotel, Jeju)

### Research Institute for Mathematical Sciences (RIMS), Japan

August 2017 – July 2025 (renewed in 2020)

- RIMS & IBS-CGP Joint Symplectic Geometry Workshop (December 2 – 4, 2019 @ Kyoto University)
- Wall-crossing Formula, Open Gromov-Witten Invariants and Related Areas (October 29 – 31, 2018 @ POSTECH)
- Pacific Rim Complex-Symplectic Geometry Conference (July 31 – August 4, 2017 @ POSTECH)

### Mathematical Research Institute (MATRIX), Australia

December 2018 – November 2021

### French-Korean International Research Laboratory (IRL) in Mathematics

January 2019 – December 2023

*This platform is expected to foster French-Korean collaboration in mathematics.*

French Members	Korean Members
Centre National de la Recherche Scientifique (CNRS) École Normale Supérieure Lyon Sorbonne Université Université de Bordeaux Université Grenoble Alpes Université de Paris Université de Strasbourg Institut polytechnique de Bordeaux	IBS Center for Geometry and Physics (IBS-CGP) Korea Advanced Institute of Science and Technology (KAIST) Korea Institute for Advanced Study (KIAS) Pohang University of Science and Technology (POSTECH) Seoul National University

## Research Highlights

**Alexander Aleksandrov : Two-dimensional gravity and a new proof of the Witten conjecture**

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**Byung Hee An : Intrinsic and extrinsic properties of graphs**

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**Jinwoo Jang : Kinetic theory of plasma in a bounded domain**

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**Jun Yong Park : Arithmetic of the Moduli of fibrations**

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**Eunjeong Lee : Finding dictionaries in geometry**

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**Yat-Hin Suen : SYZ transforms of immersed Lagrangian multi-sections**

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## Two-dimensional gravity and a new proof of the Witten conjecture

Alexander Aleksandrov

Research member since February 2017

**Quantum** gravity is one of the major challenges in natural sciences. Unification of the general relativity and quantum theory, the Holy Grail of contemporary physics, is still locked in spite of the persistent attempts of the leading scientists of XXth and XXIst centuries. There is a variety of approaches to attack this fundamental problem. One of them is to investigate the simplified toy models of the quantum gravity. These models can help us to better understand the theory of our real world.

One of the natural possibilities is to consider the models in lower dimensions. This approach often leads to a significant simplification of the master model, which still preserves some important features of the higher dimensional theory. Our world is four-dimensional, which includes the three dimensions of space and the one dimension of time. We can consider three-, two-, and one-dimensional models of quantum gravity. It is even possible to consider a zero-dimensional model, but it is trivial. One-dimensional theory possesses a few interesting properties, but still is too simple. The two-dimensional model of quantum gravity, which is simple enough to be completely solvable, is described by an extremely beautiful mathematical theory that possess a number of extraordinary properties. Two-dimensional gravity still might seem to be oversimplified, at least as a physics theory, however, it leads to a completely unpredictable relation between distant branches of pure mathematics.

The attention to the two-dimensional quantum gravity in 1980's was caused by a rapid development of the *string theory*. String theory describes, in particular, the dynamics of the strings, one-dimensional objects, which sweep two-dimensional surfaces. String theory is one of the leading candidates for the quantum theory of gravity in higher dimensions, and two-dimensional theory of gravity is an inherent part of string theory.

There exist several different physical models of two-dimensional quantum gravity. However, a variety of arguments indicate that these models should be equivalent. This led Edward Witten, a prominent American physicist, to a conjecture that unifies two *a priori* different models of two-dimensional gravity. This conjecture being formulated mathematically was extremely miraculous. Namely, one of the versions of the two-dimensional gravity, *topological gravity*, has a perfectly well-defined mathematical formulation. The function, which contains complete information about two-dimensional topological gravity, is given by a sum of the finite-dimensional integrals over the so-called *moduli spaces*. However, these moduli spaces are complicated geometrical objects, which in general makes impossible direct computations of the integrals over them.

On the other hand, according to Witten's conjecture, the same function should be described by an infinite family of non-linear equations, so-called *KdV hierarchy*. This hierarchy is named after Diederik Korteweg and Gustav de Vries, two Dutch mathematicians who applied the first equation of the hierarchy for investigation of the surface waves of long wavelength and small amplitude on shallow water in the channels. It is one of the most famous nonlinear partial differential equations. Usually nonlinearity makes it almost impossible to find enough explicit solutions of the equation. However, the KdV equation is *integrable*.

Integrability means, in particular, that there exist a complete description of all solutions of the KdV equation and all higher equations of the KdV hierarchy. This is related to the power symmetries of the integrable systems. For the KdV hierarchy these symmetries allow us to relate any two solutions of the hierarchy to each other.

This conjecture was proved by an outstanding Russian mathematician Maxim Kontsevich, who found a very explicit combinatorial description of the topological gravity. Namely, it reduces the computations on the moduli spaces to the counting of certain pictures or graphs, given by a so called Strebel-Penner ribbon graph model. This counting problem is described by a finite-dimensional integral over Hermitian matrices aka *matrix model*, which makes the calculations almost trivial. Moreover, matrix models constitute a class of the natural solutions to the integrable hierarchies. Kontsevich's proof also makes explicit the interplay between linear and non-linear equations, another characteristic property of two-dimensional gravity.

In spite of the significant progress, the relation between such different areas of mathematics as algebraic geometry, integrable systems, combinatorics and matrix models, given by the Kontsevich-Witten theory, remains largely mysterious. However, the Kontsevich-Witten theory is interesting not only because of that. Actually, it is a universal building block for a huge family of models in geometry and physics. In particular, it is a basis of the *topological recursion* - a mathematical physics construction, which appears to be surprisingly universal. It allows to compute infinite families of geometrical or physical invariants starting from simple geometrical data. Despite its significance to modern mathematics, generalization of the Kontsevich-Witten theory to other models related to two-dimensional gravity is still impossible beyond a very short list of examples. Often we can find some elements of the brilliant Kontsevich-Witten description, but usually it is not enough to complete the puzzle.

All this indicates that the Kontsevich-Witten theory requires further investigation. There are several independent proofs of the Witten conjecture/Kontsevich theorem, but on the geometric side they all use the Strebel-Penner ribbon graph model or a couple of other mathematical concepts. In their new paper, Alexander Alexandrov from IBS-CGP, Francisco Hernández Iglesias and Sergey Shadrin from KdV Institute of University of Amsterdam combine the earlier known results with the novel findings to give a completely new proof of the Witten conjecture/Kontsevich theorem. In this new proof the link between the intersection theory of the moduli spaces and integrable systems is established via the geometry of double ramification cycles.

This proof is expected to be extendable to other models of mathematical physics, related to two-dimensional gravity. This work is a part of a project, the main purpose of which is to understand the fundamental relation between the different elements of the description of two-dimensional gravity and moduli spaces. It is expected that developments in two-dimensional gravity eventually will help us to understand its four-dimensional counterpart and to describe our physical world.

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1. **Alexander Alexandrov**, Francisco Hernández Iglesias, Sergey Shadrin, *Buryak-Okounkov Formula for the  $n$ -Point Function and a New Proof of the Witten Conjecture*, International Mathematics Research Notices, 2020.

# Intrinsic and extrinsic properties of graphs

**Byung Hee An**

*Research member from March 2013 to February 2020*

## Intrinsic properties of graphs via configuration spaces

A space is basically a collection of points. However, one may think that a space is the collection of ways choosing a point in a given space. The latter viewpoint is identical to the former and looks like a tautology, but it gives us a way of generalizing a space as follows: in a given space, the collection of ways choosing two distinct points, three distinct points and so on. Then it is very useful to describe several points on a space simultaneously such as any non-collapsing motion of particles in physics.

The *configuration space* is the set consisting of all possible *configurations* (or *states*) of any number of particles and a path in the configuration space will describe a non-collapsing motion of particles. One can imagine people dancing on the floor which describes a path in a configuration space of the plane. Then what happens if two particles approach to each other? If the space they belong to is large enough, then particles can avoid each other and go their own ways. How about the case when two particles encounter each other on a single log bridge? Can they still change their position? The answer depends on whether our space has another bridge or there is a spot for one to give way to the other, which is a question about the global shape of the space. This interesting situation is due to that our space is assumed to be very restrictive, just like a graph, which is a one-dimensional singular space.

Indeed, studying configuration spaces of graphs is related with many real-life questions including robots in a factory, trains on the tracks, motion planning and so on. Rather deeply, one may find the relation between entanglements of particles over graphs, called a *braiding*, and architectures for topological quantum computers.

The configuration space of one particle over the graph is nothing but the graph itself and so the only nontrivial motion is represented by a loop of the graph. Hence we are able to detect loops from the configuration space. For two particles over the graph, we have additional nontrivial motion, called a *star move* (or *star class*), that occurs at wherever the three-way junction exists.

These two moves are elementary moves and enough to describe *one-dimensional* motions. Two- or higher-dimensional motions may exist as well even though it is not easy to imagine them. One easiest example of such motions is to consider loop and star moves simultaneously and independently at various and disjoint positions.

Mathematically speaking, these motions are explained by *homotopy invariants* such as the fundamental groups or homology groups. Hence there are a lot of canonical and interesting questions about computation, torsions, ranks, basis, finite generation, further algebraic structures, formalities and so on.

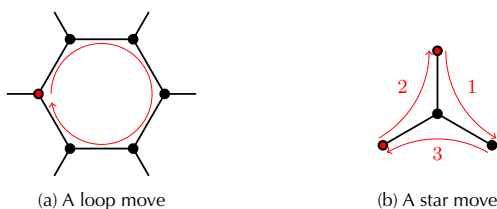


Figure 1. One- and two-particle motions over graphs

Surprisingly, homotopy invariants give us many interesting information of the underlying graph. For example, all trees can be characterized by the homotopy type of configuration spaces, and the planarity of any graph is equivalent to the torsion-freeness of the homology groups of configuration spaces of 2-or more particles.

$$T_1 \cong T_2 \iff B(T_1) \simeq B(T_2) \quad \Gamma \subset \mathbb{R}^2 \iff H_1(B(\Gamma)) \text{ is torsion-free}$$

On the other hand, the configuration spaces of graphs have an action of edges, called a stabilization which adds a particle on the desired edge. This is never possible for non-graphs and probably no one believes that the stabilization is possible since the manipulation of configuration spaces is hardly continuous. The brilliant idea that makes the stabilization real came from the discussion with CGP's former member, Gabriel C. Drummond-Cole and another collaborator, Ben Knudsen. I'm very proud that we have opened a new direction of the research and successfully figured out the edge-module structures, asymptotic behaviors and formalities as modules under the aid of the stabilization.

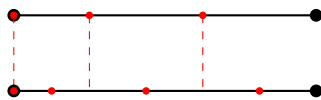


Figure 2. Stabilization on an edge by taking averages

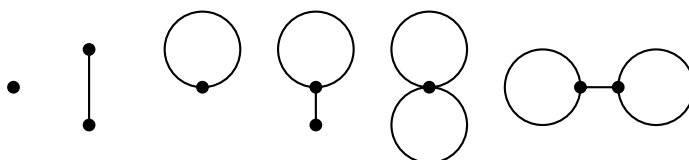


Figure 3. Complete list of edge-formal graphs

In particular, its asymptotic behavior can be captured by the growth of *betti numbers* and we eventually have a sequence of polynomials which is an invariant of the underlying graph. The degree of the *i*-th polynomial gives us an information on how tightly the graph is connected, namely, how hardly the graph can be cut by *i*-vertices. Moreover, the leading coefficient is exactly the number of ways that higher dimensional motions are formed by loops and stars. This work was published in Documenta Mathematica in 2019 and Geometry and Topology in 2020, and more interesting results are contained in four preprints posted in arXiv.

## Extrinsic properties of Legendrian graphs

Extrinsic property is a property which is determined not only by the object itself but also by the environment surrounding it. Usually, this is about the embedding of the object into a larger space and typical examples are knots and links which are embeddings of circles into three dimensional spaces. Then the rotation number or the linking number will be an example of extrinsic properties we want to see.

On the other hand, it is important to notice that the characteristic of the ambient space really does matter since it inherits to the embedded object. Namely, one can consider knots and links in various categories such as topological, smooth, or Legendrian knots and links when our ambient space is a topological, smooth or contact manifold.

*Legendrian graphs* are finite graphs embedded into the *standard contact three-sphere* whose edge satisfies the important property—Legendrian-ness. Roughly speaking, a contact three-manifold is a smooth manifold together with a hyperplane field, which never give us a tangent bundle of any surface, and being Legendrian means lying on hyperplanes.

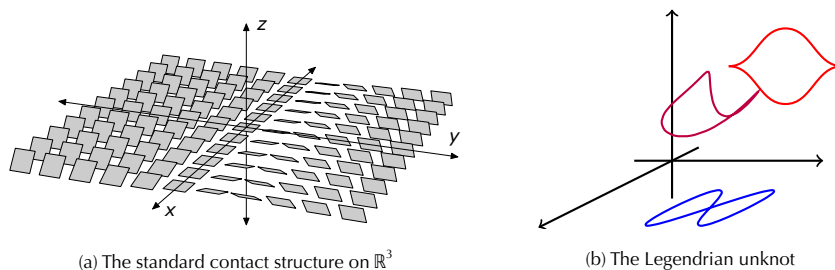


Figure 4. A contact structure and a Legendrian unknot

Legendrian graphs appear in the dynamics of a *Weinstein domain* of dimension four. The infinite boundary of Weinstein domain is a contact manifold and the *core* consisting of points that never diverge intersects the infinite boundary along a singular Legendrian locus, namely, a Legendrian graph. In particular, when the given domain is the four-ball, then the dynamics is completely determined up to homotopy by the Legendrian graph at the infinite boundary—the contact three-sphere.

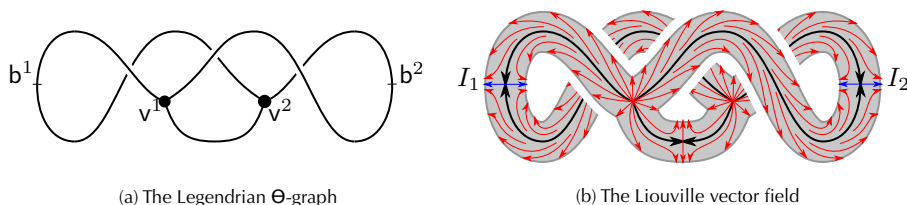


Figure 5. The Legendrian  $\Theta$ -graph and the Liouville vector field

(One of) The most important and interesting information of a Weinstein domain is the *wrapped Fukaya category*, which is a beautiful package of the following: i) certain subspaces called Lagrangians; ii) their intersections; iii) their behavior at infinity called the *wrapping*. For a Weinstein four-ball, the wrapped Fukaya category will be determined by the Legendrian graph at infinity for any Weinstein four ball as mentioned above.

What kind of invariants are known for Legendrians? For Legendrian knots and links, the rotation numbers and self-linking numbers are known to be invariant, called *classical invariants*. It is very interesting to ask whether every Legendrian link can be classified by classical invariants and the underlying link type, but unfortunately, the answer is no. Hence more powerful (or sensitive) invariants is needed and we actually have such invariants, called *modern invariants*, including algebraic, combinatorial and sheaf theoretic invariants.

The Chekanov-Eliashberg's DGA(=differential graded algebra) is an algebraic invariant which assigns a DGA to each Legendrian link. This is motivated by the bigger machinery, called the *symplectic field theory* and is really powerful. It also has many recipes to cook up such as augmentations and (filtered/linearized) homologies, and moreover, it is conjectured that the DGA is equivalent to the wrapped Fukaya category. This is really nice but it was not known how this DGA invariant is extended to Legendrian graphs.

CGP's former member Youngjin Bae and myself found the way. If I remember correctly, it was Spring 2016 when I knocked the door of Youngjin's office and said "let's do this project." At the beginning, it was just an extension of the previous project on Legendrian singular knots joint with Youngjin and Seonhwa Kim, and we never knew that the project is actually a tip of an iceberg.

The project extending DGAs to Legendrian graphs kept growing as our vision is getting wider. It also includes rulings, sheaves, Lagrangian fillings, mirror symmetries, and we are now working on cluster algebras together with Eunjeong Lee (CGP) and going further. The result on the extension of the Chekanov-Eliashberg's DGA construction for Legendrian graphs is published in Journal of Topology in 2020 consisting of 93 published pages, and more results on various invariants are submitted for publications in three papers.

## Seven years in CGP

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When I joined CGP, the work assigned to me was not only research but also management of the IT system and assets of the Center. For the first two years, I could hardly focus on my research because I should build up our servers, websites and a managing system. It was such a hard task and it seemed impossible to do it alone. Frankly speaking, this was the most painful time for me of the seven years I had worked there, but there were many good people who cheered me up. Looking back now, I think I grew up the most at this period.

After that, I could spend more time on my research with the help of the IT assistants, Jaekwan Kim, Jaeheung Park and Sujin Park. I had started research collaboration with CGP members – Youngjin Bae, Seonhwa Kim, Yunhyung Cho and Gabriel C. Drummond-Cole. I could learn a lot of new things from them and they actually made me grow up as a researcher.

These guys left CGP before me. I wished that I could stay longer in CGP but I had to leave too.

Eleven published papers, three preprints, many good friends and most of all, two children (three in total). These are what I have achieved during I was in CGP. If someone thinks that I am productive, I want to say that it's because I was healthy and the place where I worked was full of good people and opportunities to upgrade myself.

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# Kinetic theory of plasma in a bounded domain

*An  $L^2$ - $L^\infty$  approach for the Landau kinetic equation in a bounded domain*

**Jin Woo Jang**

*Research member from September 2016 to February 2020*

## References

1. **Jin Woo Jang** et al., *The Landau equation with the specular reflection boundary condition*, *Archive for Rational Mechanics and Analysis*, 236, 3, 1389–1454, 2020.

One part of the huge field of nonequilibrium statistical physics is in the mathematical theory of collision process of dilute gases and plasmas. Hot plasmas or gases suffer collisions when the effective collision frequency (or the reciprocal of the mean-free-time of a particle) is much larger than the frequency of variation of the external or the self-consistent fields or when the field wavelength is larger than the particle mean-free-path. In this case, the effect from the collisions is dominating the background field effects and the dynamics can be described by the collisional kinetic equations such as the Boltzmann equation. The prototype of the Boltzmann equation was introduced by James Clerk Maxwell and has later been derived by Ludwig Boltzmann in 1872. It is a fundamental model for rarefied gases taking collisions into account. One of the most important questions in the mathematical analysis of the kinetic theory and the statistical physics is not just on the global well-posedness but also on whether the particle system converge to an equilibrium state or not. If not, what would the long-time asymptotics of the solution look like? These questions have never been proved rigorously for a generic Cauchy problem with generic initial-boundary conditions and are highly open.

The study of kinetic equations is physically motivated by classical mechanics with large number of interacting particles whose dynamics can further be described in a statistical and mesoscopic perspective. The type of particles here can be very diverse. Equations for classical or quantum particles can be derived via appropriate assumptions of Newtonian or quantum physics, respectively. On the other hand, one can also consider special relativistic kinetic equations that take into account Einstein's special relativity for fast moving particles whose speed is comparable to the speed of light. The nature of a Cauchy problem in kinetic theory can also depend on the physical phase-space that the particles reside. Namely, the nature of the problem varies on whether one considers particle dynamics in an unbounded domain like particles in the universe or one consider the dynamics of particles in a bounded domain like plasmas in Tokamak or Stellarator. In addition, there are various mathematical problems depending on whether or not external fields or external obstacles are present.

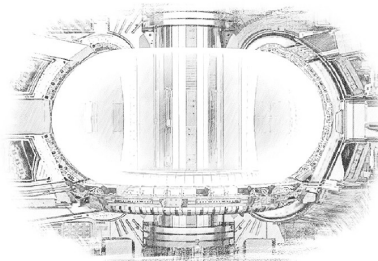


Figure 1. Hot plasmas in a tokamak under a nuclear fusion process [ITER Organization]

## The Landau equation in a bounded domain

Especially, if one starts considering a physically relevant particles like electrons, protons, atoms, or molecules, one must consider the collision operator which has been derived from the Coulombic potential. Unfortunately, the classical Boltzmann equation is not well-defined for the physical Coulombic interactions though the interactions between electrons and protons follow the Coulombic potential. Via the consideration of a modified potential called Debye or Yukawa potential, Lev Landau in 1936 was able to derive the Boltzmann operator for the modified Coulombic potential, which decays like  $1/r$  until the screening distance and decays exponentially fast beyond the screening distance.

The Landau kinetic equation is given by  $\partial_t F + v \cdot \nabla_x F = Q(F, F)$ , where the unknown  $F = F(t, x, v)$  is a non-negative function which represents the density distribution of particles in the phase space for each time  $t \geq 0$ . The phase space consist of the spatial domain  $\mathcal{Q} \subset \mathbb{R}^3$  and the domain for velocities  $\mathbb{R}$ . The Landau collision operator  $Q(F, G)(v)$  is defined as

$$\nabla_v \cdot \left\{ \int_{\mathbb{R}^3} \phi(v-v') [F(v') \nabla_v G(v) - G(v) \nabla_v F(v')] dv' \right\},$$

where the collision kernel for the Coulombic particle interactions

$$\phi(z) \stackrel{\text{def}}{=} \left\{ I - \frac{z}{|z|} \otimes \frac{z}{|z|} \right\} \cdot |z|^{-1},$$

is a symmetric and non-negative matrix such that  $\phi_{ij}(z)z_i z_j = 0$ .

Unfortunately, for a gas-wall interaction case (a problem in a bounded domain), the decay of solutions to a global equilibrium has never been obtained for the Landau equation due to the lack of control on the derivative norm  $\|f(t)\|_{L_x^\infty W_v^{1,\infty}}$  of the probabilistic density function  $f$  for any time  $t > 0$ . In the case of the Boltzmann equation where we do not need to control the derivative norm, we have a good amount of mathematical proofs in a convex or nonconvex domain.

In a joint work with three other mathematicians (Y. Guo, H. J. Hwang, and Z. Ouyang), the author provided the first mathematical proof [1] for the global wellposedness and the decay to Maxwellian for the Landau equation in a general 3-dimensional smooth bounded domain with the specular reflection boundary condition via the  $L^2$ - $L^\infty$  approach in the diagram (Figure 2) for the linearized Landau equation nearby the Maxwellian equilibrium.

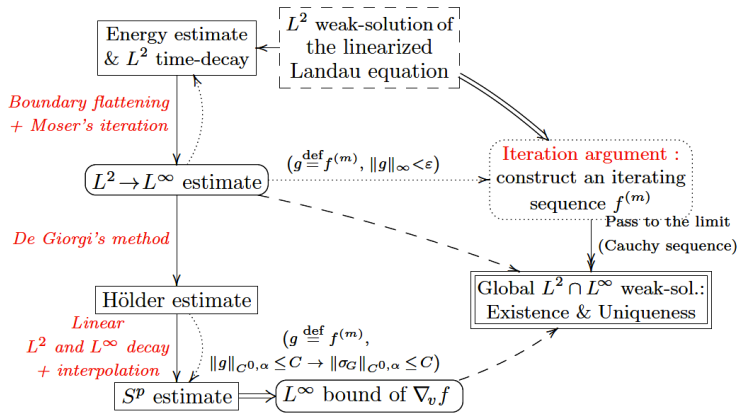


Figure 2. An  $L^2$ - $L^\infty$  approach [Springer-Verlag GmbH Germany]

The main obstacle to accomplish the main  $L^2$ - $L^\infty$  approach in the diagram is from the boundary, as in a bounded domain, the solutions cannot be smooth up to the grazing set even though we have the diffusion term in  $v$  variable. Hence some new mathematical tools involving much weaker norms must be developed. One of the main ideas to treat this problem is to take some advantages from the boundary flattening and the extension of the local domain beyond the boundary by taking the mirror reflection. We have shown that the solutions are Hölder continuous across the boundary in the specular reflection boundary case.

Once, we extend the domain near the boundary, we can now consider the local domain as a local bounded set in the whole space, where we can keep arguing on the local  $L^\infty$ -Hölder continuity bootstrapping and the Morrey-type inequalities to complete the diagram. The holy grail of our techniques is from the fact that there exists a specific choice of boundary flattening mapping and the mirror extension formula so that the transformed equation is now in the form of the linear Fokker-Planck equation with a generic coefficients

$$\partial_t \bar{f} + w' \cdot \nabla_{y'} \bar{f} = \nabla_{w'} \cdot (\mathbb{A} \nabla_{w'} \bar{f}) + \mathbb{B} \cdot \nabla_{w'} \bar{f} + \mathbb{C} \bar{f},$$

where the coefficients  $\mathbb{A}$ ,  $\mathbb{B}$ , and  $\mathbb{C}$  are piecewise-defined and  $\mathbb{A}$  is further Hölder continuous across the boundary.

## Living in Bonn, Germany



It was the last summer in 2019 when I first visited the city of Bonn in Germany. That was for the participation as a visiting scholar in the Junior Trimester Program in Kinetic Theory held by Hausdorff Institute for Mathematics for about two months. I actually had a family trip to Germany when I was really young, but I don't remember much from it since I was too young at the moment. The entry at the airport was very convenient for a Korean citizen who doesn't even speak any German. I remember that the first curry-sausage (currywurst) that I had was very impressive, and I've realized that the curry-sausage with some fries is a common meal for German people.

Last summer in 2019 was one of the hottest summers in German history; at least people who told me about it would have felt in that way. One of the main reasons that it is relatively difficult to endure summer here is because there is no air conditioner in almost every building in Bonn. I also think that the situation is the same in the whole country and several other countries nearby. I felt it so weird and asked to my former officemate who was from Vienna, Austria. According to what he told me, there seem to be two main reasons behind the "no air conditioner at all" situation in Europe. Namely, he said many people in Europe think that it is not too bad to endure the hot weather in the summer, since there are only very few days in the summer during which it is extremely hot that nobody could endure against.

The other reason according to him was that people want to preserve the beauty of the buildings. Namely, all the buildings are being preserved for the decades up to several hundreds of years, and they think that the outdoor fan for the air conditioner at each window and each unit of the apartment would harm the unity of the classic and antique beauty. Indeed, there was no air conditioner everywhere except for one classroom for the conference and the markets. Even hotels didn't have any air conditioners. My office had a wooden shade attached to the



outer parts of the windows, and the sun block was quite effective. However, there were actually a week of flame last summer and I had to go out of the building and sit under the shade of a tree in front of my office. But since I was free of

any usual cold/sickness by the strong air conditioner in the summer, I also think it is not too bad to live without an air conditioner now.



The building for the math department at the University of Bonn was magnificent; the size of a building is comparable to the size of a huge department store in Korea and it was amazing that the whole building was used only for mathematics. The department has a subdivision of the following four mathematical institutes: Mathematical Institute, Institute of Applied Mathematics, and Institute of Numerical Simulation, and Institute of Discrete Mathematics.

Several groups in the Institute of Applied Mathematics that I belong to have had lunch together everyday at 12:45, which I really liked much. Everyday it was very enjoyable that a group of about 15 people have lunch together. Unless there are some other special issues, we gathered together and went to have lunch. I believe that it is indeed great to have such natural communications with other scholars who work in the same field, since those natural conversations can lead us to a better understanding in our expertise, can lead us to a collaboration, or at least can build a strong bond among the scholars. Some professors who usually have very light lunch have also joined almost everyday and do math on a piece of napkin waiting for other people finishing their meals, which impressed me a lot.



However, this was only for the first month, March 2020. After this, it has been prohibited to gather together like this since April of 2020 due to the unexpected coronavirus pandemic. Until the summer, all the meetings and the courses have been done via the Zoom online meetings. It seems that nobody expected it to be this long. It is already in the mid-October, and today Germany renewed its maximum number of confirmed cases per day again: more than 8,000 new cases today. I hope that the pandemic will end soon and wish lots of health to everyone over the world.

## Arithmetic of the Moduli of fibrations

**Jun Yong Park**

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**A**rithmetic geometry arose as a beautiful theory unifying geometry and number theory where we are interested in the geometry and topology of algebraic varieties, which are roughly speaking solution sets of polynomial equations. Through the famous Weil conjectures, proven by Deligne, we can get information about geometric & topological, and more precisely cohomological aspects of those equations by counting their solutions over finite fields. The algebraic varieties we consider are often moduli spaces, that are the parameter spaces for certain objects such as Abelian varieties or algebraic curves and surfaces. These objects arise in different contexts notably in representation theory, number theory and mathematical physics and counting them often leads to surprising connections between those seemingly different fields. Recently, we have focused on a perspective called number theory over function fields, which allows us to the study of classical problems in number fields realm, translated to the (global) function fields realm.

Concretely, we start by expressing a problem in number theory as the problem of finding an approximate formula (or proving an already conjectured one) for the number of elements in a fixed set. Many problems in analytic number theory and arithmetic have this form or can be reduced to it. If this problem seems intractable, we can consider a finite field analogue, which may be easier. We take the definition of the set we wish to count and modify it by replacing each occurrence of the integers in the definition with the ring  $\mathbb{F}_q[t]$  of polynomials over a finite field, but otherwise change it as little as possible.

We can almost always recognize this new set as the set of points of an algebraic variety defined over  $\mathbb{F}_q$ , i.e., the solutions in  $\mathbb{F}_q$  of some finite list of polynomial equations, possibly after removing the solutions to another list of polynomial equations. The Grothendieck-Lefschetz trace formula then gives an approximate formula for the number of points in this new set if we can calculate the compactly supported, étale cohomology of this algebraic variety. If we do not know how to calculate this cohomology, we could transfer the problem to yet another setting. We can consider the analogous, purely topological problem, of calculating the compactly supported, singular cohomology of the space of solutions in  $\mathbb{C}$  to the same list of polynomial equations. This can be attacked using any of the many known topological methods for computing cohomology groups. Once this is done, we can sometimes transfer back to the original finite field problem using comparison results for cohomology, or by using finite field analogues of our topological techniques.

To illustrate, one of the fundamental problems of the number theory is the enumeration of number fields.

**Problem 1.** How many number fields are there of degree  $d$  over  $\mathbb{Q}$  and discriminant  $\leq \mathcal{B}$  ?

Since the classical finiteness theorem of Hermite–Minkowski, there have been many remarkable results giving the upper bounds as in Schmidt, Ellenberg and Venkatesh. One could consider a one-dimensional analogue of this problem, which concerns counting the arithmetic curves over a number field.

Geometrically, this is the study of *fibration*, a flat proper surjective morphism  $f : X \rightarrow C$  of an algebraic surface  $X$  over a smooth complete curve  $C$ , lies at the heart of the birational classifications of surfaces over any algebraically closed fields. The enumeration of arithmetic curves over an algebraic number field is a central problem in number theory constituting a crucial part in the Gerd Faltings' celebrated proof of the Mordell's conjecture in 1983 by Faltings. In this regard, the finiteness of the number of *non-isotrivial* smooth families of algebraic curves over  $\mathbb{Q}$  with bounded bad reduction was a problem first proposed by Igor R. Shafarevich in his 1962 address at the International Congress in Stockholm.

**Problem 2** (Shafarevich's problem for arithmetic curves). Let  $S$  be a finite set of places of a number field  $K$ . How many distinct  $K$ -isomorphism classes of curves  $X/K$  are there, of genus  $g \geq 2$  and possessing good reduction at all primes  $P \notin S$  ?

'Shafarevich's conjecture' is the assertion that there is only a finite number for any given  $(g, K, S)$  (i.e., *Finiteness principle for curves*). We consider a problem of analogous nature where the number field  $\mathbb{Q}$  is replaced by the global function field  $\mathbb{F}_q(t)$  through the *global fields analogy* that is based on the fundamental Artin-Whaples Theorem proven in 1945 which emphasized the close analogy between the theory of algebraic number fields and the theory of function fields of algebraic curves over finite fields.

**Theorem 3** (Artin-Whaples Theorem). *Every global field is a finite extension of  $\mathbb{Q}$  or  $\mathbb{F}_q(t)$ .*

Using techniques from birational geometry, algebraic stacks, and arithmetic geometry, we study explicit examples of moduli of fibrations, fibered by stable elliptic & hyperelliptic curves and principally polarized Abelian surfaces over global function fields. Firstly, we effectively prove the geometric Shafarevich's conjecture for semistable elliptic curves over  $\mathbb{P}_{\mathbb{F}_q}^1$  with a marked Weierstrass section, which admit squarefree discriminants. We define the counting function  $Z_{1, \mathbb{F}_q(t)}(\mathcal{B})$  for a positive real number  $\mathcal{B}$  as follows:

$$Z_{1, \mathbb{F}_q(t)}(\mathcal{B}) := |\{\text{Semistable elliptic curves over } \mathbb{P}_{\mathbb{F}_q}^1 \text{ with } 0 < ht(\Delta(X)) = q^{12n} \leq \mathcal{B}\}|$$

Then, we acquire the following descriptions of  $Z_{1, \mathbb{F}_q(t)}(\mathcal{B})$  where  $\mathcal{O}_q$ -constant is an explicit rational function of  $q$  by bounding the *height* of discriminant  $\Delta_1(X)$ :

**Theorem 4.** *If  $\text{char}(\mathbb{F}_q) \neq 2, 3$ , then the function  $Z_{1, \mathbb{F}_q(t)}(\mathcal{B})$ , which counts the number of semistable elliptic curves over  $\mathbb{P}_{\mathbb{F}_q}^1$  ordered by  $0 < ht(\Delta(X)) = q^{12n} \leq \mathcal{B}$ , satisfies:*

$$Z_{1, \mathbb{F}_q(t)}(\mathcal{B}) \leq 2 \cdot \frac{(q^{11} - q^9)}{(q^{10} - 1)} \cdot \left(\mathcal{B}^{\frac{5}{6}} - 1\right)$$

*which is an equality when  $\mathcal{B} = q^{12n}$  with  $n \in \mathbb{N}$  implying that the acquired upper bound is a sharp estimate where the leading term is of order  $\mathcal{O}_q\left(\mathcal{B}^{\frac{5}{6}}\right)$  and the lower order term is of order  $\mathcal{O}_q(1)$ .*

Given a function  $f : \mathbb{N} \rightarrow \mathbb{R}$  with coefficients depending on an integer  $q$ , a sharp estimate of a function  $f$  (with coefficients depending on  $q$ ) means an upper bound  $g$  (with same properties) that coincide with  $f$  on infinitely many values of  $x \in \mathbb{N}$ . We formulate the following heuristic on  $\mathcal{Z}_{1,\mathbb{Q}}(\mathcal{B})$  by passing the above sharp estimate through the global fields analogy:

**Conjecture 5.** The counting  $\mathcal{Z}_{1,\mathbb{Q}}(\mathcal{B})$  of semistable elliptic curves over  $\mathbb{Z}$  by  $0 < ht(\Delta) \leq \mathcal{B}$  follows from the sharp estimate counting on  $\mathcal{Z}_{1,\mathbb{F}_q(t)}(\mathcal{B})$  through the global fields analogy. Namely,  $\mathcal{Z}_{1,\mathbb{Q}}(\mathcal{B})$  has the leading term of order  $\mathcal{O}\left(\mathcal{B}^{\frac{5}{6}}\right)$  and the lower order term of order  $\mathcal{O}(1)$ .

On a related note, the number of discriminants  $\Delta_1$  of an elliptic curve over  $\mathbb{Z}$  with smooth generic fiber such that  $\Delta_1 \leq \mathcal{B}$  is estimated to be asymptotic to  $\mathcal{O}\left(\mathcal{B}^{\frac{5}{6}}\right)$  by Brumer and McGuinness. The lower order term of order  $\mathcal{O}\left(\mathcal{B}^{(7-\frac{5}{27}+\epsilon)/12}\right)$  for counting the stable elliptic curves over  $\mathbb{Q}$  by the bounded height of squarefree  $\Delta_1$  was suggested by the work of Baier. In fact, Baier proved his asymptotic under the assumption of the generalized Riemann hypothesis with the twelfth root of the naïve height function on elliptic curves; this gives arise to the prediction above. We note that counting curves over a global field  $K$  by height of discriminant  $ht(\Delta_g)$  is more difficult than counting by naïve height. In the case of counting elliptic curves ordered by height of discriminant  $ht(\Delta_1)$ , the region of lattice points with bounded discriminant has *cusps*, meaning that there are *unbounded points* with large  $a_4, a_6 \in \mathcal{O}_K$  (i.e., elliptic curves with large naïve height) and small discriminant  $\Delta_1$ . Controlling these cusps is difficult, even if the ABC conjecture is assumed.

By  $\ell$ -adic Leray spectral sequence with respect to faithfully flat evaluation morphism  $ev_\infty : \text{Hom}_n(\mathbb{P}^1, \mathcal{P}(a, b)) \rightarrow \mathcal{P}(a, b)$ , we compute  $\ell$ -adic étale Betti numbers  $\dim_{\mathbb{Q}_\ell} \left( H_{\text{ét}}^i(\text{Hom}_n(\mathbb{P}^1, \mathcal{P}(a, b))_{/\overline{\mathbb{F}}_q}; \mathbb{Q}_\ell) \right)$  showing  $\mathbb{Q}_\ell$ -vector spaces are all one dimensional for  $i = 0, 3$  and vanishes for all other  $i$  implying that the  $\ell$ -adic rational cohomology type of  $\text{Hom}_n(\mathbb{P}^1, \mathcal{P}(a, b))$  through the Grothendieck-Lefschetz trace formula:

**Corollary 6.** Hom stack  $\mathcal{L}_{1,12n} := \text{Hom}_n(\mathbb{P}^1, \overline{\mathcal{M}}_{1,1})$  with  $n \in \mathbb{N}$  over  $\overline{\mathbb{F}}_q$  with  $\text{char}(\overline{\mathbb{F}}_q) \neq 2, 3$  isomorphic to the moduli stack of stable elliptic fibrations over  $\mathbb{P}^1$  with  $12n$  nodal singular fibers and a marked Weierstrass section has the following compactly supported  $\ell$ -adic étale cohomology and Galois representations of mixed Tate type

$$H_{\text{ét},c}^i(\mathcal{L}_{1,12n/\overline{\mathbb{F}}_q}; \mathbb{Q}_\ell) \cong \begin{cases} \mathbb{Q}_\ell(- (10n + 1)) & i = 20n + 2, \\ \mathbb{Q}_\ell(- (10n - 1)) & i = 20n - 1, \\ 0 & \text{else.} \end{cases}$$

By the Poincaré duality, the ordinary  $\ell$ -adic étale cohomology is equal to

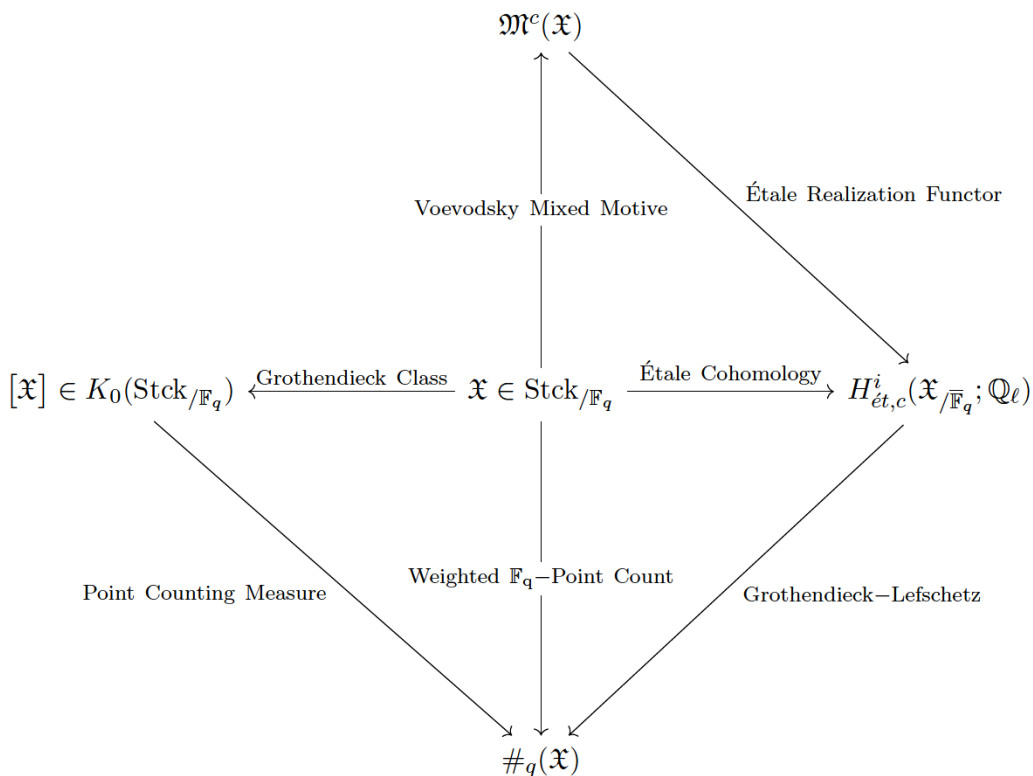
$$H_{\text{ét}}^i(\mathcal{L}_{1,12n/\overline{\mathbb{F}}_q}; \mathbb{Q}_\ell) \cong \begin{cases} \mathbb{Q}_\ell(0) & i = 0, \\ \mathbb{Q}_\ell(-2) & i = 3, \\ 0 & \text{else.} \end{cases}$$

Our main theorem unifies and generalizes these results, showing the compactly supported motive  $\mathfrak{M}^c(\mathrm{Hom}_n(\mathbb{P}^1, \mathcal{P}(a, b)))$  when  $\mathrm{char}(K) \nmid a, b$  in Voevodsky’s triangulated category  $\mathbf{DM}(K, \mathbb{Q})$  of mixed motives with  $\mathbb{Q}$ -coefficients lies in the subcategory  $\mathbf{DTM}(K, \mathbb{Q})$  of mixed Tate motives.

**Theorem 7.** *Let  $K$  be a field with  $\mathrm{char}(K) \nmid a, b$ . Then  $\mathrm{Hom}_n(\mathbb{P}^1, \mathcal{P}(a, b))$  is of Tate type in  $K_0(\mathrm{Stck}/K)$  and the compactly supported motive is of mixed Tate type, i.e.*

$$\mathfrak{M}^c(\mathrm{Hom}_n(\mathbb{P}^1, \mathcal{P}(a, b))) \in \mathbf{DTM}(K, \mathbb{Q}) \subset \mathbf{DM}(K, \mathbb{Q}).$$

The following diagram with  $\mathfrak{X} = \mathrm{Hom}_n(\mathbb{P}^1, \mathcal{P}(a, b))$  summarizes the above discussion.



## References

1. Changho Han and **Jun Yong Park**, *Arithmetic of the moduli of semistable elliptic surfaces*, *Mathematische Annalen*, Vol. 375, No. 3–4, 1745–1760, 2019.
2. **Jun Yong Park** and Hunter Spink, *Motive of the moduli stack of rational curves on a weighted projective stack*, *Research in the Mathematical Sciences*, Special Issue of PIMS 2019 Workshop on Arithmetic Topology, Vol. 8, No. 1, 2021.

# Finding dictionaries in geometry

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## Dictionary in equivariant algebraic geometry

There are many areas in mathematics to which topology, geometry, representation theory, and combinatorics are interwoven in a very beautiful way. For instance, in toric geometry, which studies geometric objects having fruitful symmetries, there is a bijective correspondence relation between the family of projective toric varieties and the family of full dimensional lattice polytopes.

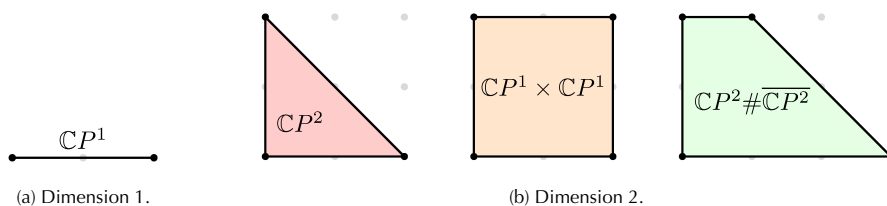


Figure 1. Some examples of lattice polytopes and the corresponding toric varieties.

It provides a dictionary between a family of projective toric varieties and that of polytopes, that is, a lot of geometric and topological properties of a projective toric variety are read off from the combinatorics of the corresponding polytope. On the other hand, the representation of a compact torus arises as a space of holomorphic sections of line bundles over smooth projective toric varieties, and the character of the representation is encoded in lattice points inside of the polytope.

One another absorbing example is the Borel–Weil–Bott theorem, which provides a connection between the geometry of homogeneous spaces and the representations. More precisely, it shows how a family of irreducible representations can be obtained from holomorphic sections of certain complex vector bundles over flag varieties. On the other hand, Guillemin and Sternberg introduced a completely integrable system on the flag manifold, and this leads us to the Gelfand–Cetlin polytope which provides a combinatorial model for parametrizing basis of irreducible representations. One notable result is that a detailed description of the topology of Gelfand–Cetlin fibers is given by the combinatorics of the Gelfand–Cetlin polytope.

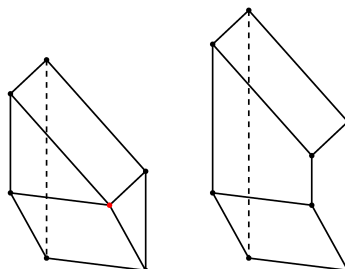


Figure 2. A Gelfand–Cetlin polytope (left) and its small Fano toric desingularization (right).

## Theory of Newton–Okounkov bodies

Recently, the theory of Newton–Okounkov bodies have been rapidly developing with the attempt to enlarge this fruitful connection. Newton–Okounkov body theory constructs a convex body for an algebraic variety with a given very ample line bundle. This construction enlarges the known beautiful relation arisen in toric geometry, and moreover, flag varieties and Gelfand–Cetlin polytopes. More precisely, if the Newton–Okounkov body is a rational convex polytope, Anderson proved that there is a flat toric degeneration of a given algebraic variety whose special fiber is the *toric* variety defined by the Newton–Okounkov polytope.

By associating a projective variety with a projective *toric* variety, one could use many tools developed in toric geometry to study the (not necessarily toric) projective variety. Indeed, Harada and Kaveh produce a completely integrable system on a given algebraic variety whose image is the Newton–Okounkov polytope. By the pioneering work of Nishinou–Nohara–Ueda, the toric degeneration is used to compute the symplectic-geometric data, so-called *the Floer disk potential*, of a Lagrangian submanifold of the given variety determined by the completely integrable system in terms of the combinatorics of the polytope, like as the case of toric varieties. Here, there are certain technical assumptions to apply their result, for example, the toric variety has to admit a small Fano toric desingularization.

## Flag varieties and their Newton–Okounkov bodies

One of the most interesting examples of Newton–Okounkov bodies is a family of string polytopes, which are introduced by Littelmann as a generalization of the Gelfand–Cetlin polytopes. These polytopes provide a combinatorial understanding of crystal basis in finite-dimensional representations.

There exist a bunch of different string polytopes so that one can construct different toric degenerations of flag varieties. Therefore, it would be interesting to study **which topological or geometric data can be obtained from different toric degenerations and how these data are related to each other**. Because a very few are known for the combinatorics of these polytopes, one still does not overcome previously described technical issues to obtain symplectic-geometric data of each toric degenerations. Accordingly, it is important to study combinatorics of these Newton–Okounkov bodies of flag varieties.

Collaborating with researchers in the Center for Geometry and Physics and other mathematicians, Eunjeong Lee provides a complete classification of the string polytopes which are combinatorially equivalent to the Gelfand–Cetlin polytope and studies enumeration of such string polytopes. Furthermore, she provides a concrete construction of small Fano toric desingularization for some specific cases. Consequently, the corresponding Floer theoretical disk potential of the Lagrangian submanifold has been computed in these cases.

$n$	1	2	3	4	5	6	7
# of Gelfand–Cetlin type string polytopes	1	2	6	40	916	102176	68464624
# of string polytopes	1	2	16	768	292864	1100742656	48608795688960

Table 1. The first few terms of the number of Gelfand–Cetlin type string polytopes and that of all string polytopes.

There is another method of constructing toric degenerations of projective varieties. According to the revolutionary work of Gross–Hacking–Keel–Kontsevich, the theory of cluster algebras provides a different approach to toric degenerations of compactified cluster varieties, and these toric degenerations are related by considering *mutations* on the corresponding polytopes. Recently, Fujita and Oya study Newton–Okounkov bodies of flag varieties from their cluster structures. In the case of flag varieties, it is provided that string polytopes arising from representation theory are these bodies. We hope to get one step closer to answering the preceding question not only for the flag varieties but also for other algebraic varieties using the theory of cluster algebras.

## Publications

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1. Yunhyung Cho, Yoosik Kim, **Eunjeong Lee**, and Kyeong-Dong Park, *On the combinatorics of string polytopes*, arXiv:1904.00130, submitted.
2. Yunhyung Cho, Yoosik Kim, **Eunjeong Lee**, and Kyeong-Dong Park, *Small resolutions of toric varieties associated to string polytopes of small indices*, arXiv:1912.00658, submitted.
3. Yunhyung Cho, Jang Soo Kim, and **Eunjeong Lee**, *Enumeration of Gelfand–Cetlin type reduced words*, arXiv:2009.06906, submitted.

# SYZ transforms of immersed Lagrangian multi-sections

**Yat-Hin Suen**

Research member since August 2018

## Mirror Symmetry

String theory attempts to unify general relativity and quantum field theory. The theory suggests that our fundamental particles are string-like objects, either a loop (closed string) or a segment (open string). Different particles arise as these strings vibrate differently. For (super-)string theory to work, it turns out that 6 extra hidden dimensions are needed in the universe. In other words, there are in total 9 dimensions of space and 1 dimension of time. According to string theory, these extra 6 dimensions curl up in the 10 dimensional spacetime and they form a special mathematical object called *Calabi-Yau manifolds*. By the famous theorem of Shing-Tung Yau, Calabi-Yau manifolds are vacuum solutions to the Einstein Field Equations and string theory suggests that the physical laws of a universe are completely determined by the shape of the corresponding Calabi-Yau manifold. Later, physicists discovered that two geometrically or even topologically different Calabi-Yau manifolds can give rise to the same set of physical laws. This phenomenon is known as mirror symmetry. Mathematically speaking, mirror symmetry is an equivalence between *symplectic geometry* (A-model) and *algebraic geometry* (B-model). In around 1990, by mirror symmetry, Candelas, de la Ossa, Green and Parkes gave a prediction of the number of rational curves inside a generic quintic Calabi-Yau 3-fold, which is a long-standing conjecture in enumerative geometry. This discovery caught the attention of the mathematical society immediately.

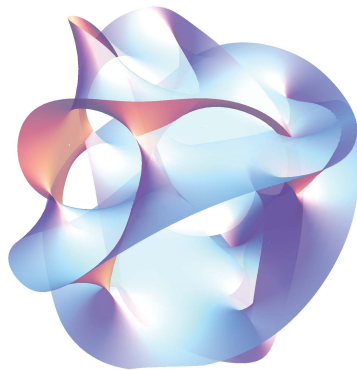


Figure 1. A Calabi-Yau manifold [math.wikia]

In 1994, Kontsevich proposed a mathematical definition of mirror symmetry, which is now known as the *homological mirror symmetry* (HMS). He formulated mirror symmetry as an equivalence between two categories. For A-model, one should look at the *Fukaya category*. An object in the Fukaya category is a *Lagrangian brane*, and the morphism space between two Lagrangian branes is given by the *Floer cohomology* between them. Physically, Lagrangian branes are the boundary conditions for the endpoints of an open string and the Floer cohomology encodes the number of ways that open strings can vibrate between two Lagrangian branes. For B-model, there is a well-studied category, called the *derived category of coherent sheaves*.

An object there is a *coherent sheaf* and the morphism space between two coherent sheaves is given by the so-called *extension groups*. HMS states that there is a (quasi-)equivalence between the Fukaya category of a Calabi-Yau manifold and the derived category of its mirror partner.

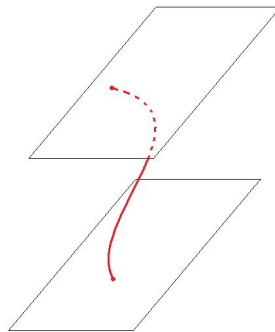


Figure 2. An open string with boundary on two Lagrangian branes [WordPress]

Two years after Kontsevich's proposal, Strominger, Yau and Zaslow proposed an entirely geometric way in understanding mirror symmetry, which is commonly known as the SYZ conjecture. They suggested that mirror symmetry is *T-duality*. This provides a geometric way in construct mirror pairs and to prove HMS. The SYZ proposal suggested that a Calabi-Yau manifold should admit a *special Lagrangian torus fibration* and its mirror is obtained by taking dual torus fibration. Furthermore, symplecto-geometric information can be interchanged with algebro-geometric information via a Fourier-Mukai-type transform, which is called the *SYZ transform*. The SYZ transform has been successful to construct mirror pairs and prove HMS beyond Calabi-Yau cases.

## SYZ transform and its impact

Together with Kwokwai Chan, I studied how SYZ transform can be used to get a deeper understanding on immersed Lagrangian branes — more specifically, immersed Lagrangian multi-sections [1]. An immersed Lagrangian brane is a Lagrangian brane that can self-intersect and a Lagrangian multi-section is a Lagrangian brane that covers the base of the fibration several times. It is widely believed that mirror of Lagrangian multi-sections are holomorphic vector bundles. In the case which the covering is unbranched, they successfully produced a holomorphic vector bundle on the mirror manifold via SYZ transform. Furthermore, we studied the relation between Lagrangian surgery and vector bundle extension. Lagrangian surgery is a method in symplectic geometry to remove intersection points and produce a new Lagrangian brane, while vector bundle extension is a classical way in algebraic geometry to produce a vector bundle from two vector bundles. We proved that on a 2-torus, if one restricts the attention to immersed Lagrangian multi-sections that are “stable”, then Lagrangian surgery and vector bundle extension correspond to each other via SYZ transform.

It is well-known that Floer cohomology between two Lagrangian branes is invariant under a special move called *Hamiltonian isotopy* inside the symplectic manifold. Therefore, Hamiltonian isotopic Lagrangian branes should be regarded as the same object in the Fukaya category, and hence should give rise to the same mirror object.

Based on the study of surgery-extension correspondence on the 2-torus, we discovered that, when immersed objects are considered, there are non-Hamiltonian isotopic immersed Lagrangian multi-sections that can give rise to same mirror object.

To explain this phenomenon, we study the immersed Floer theory of a general immersed Lagrangian brane in a general compact symplectic manifold. We introduced a new notion called the *lifted-Hamiltonian isotopy* and proved that the Floer theory of an immersed Lagrangian brane stays the same after such move.

As the Floer theory is invariant, these two objects should be regarded as the same in the immersed Fukaya category and thus, should share the same mirror object. The strange phenomenon on the 2-torus can then be explained by showing that two immersed Lagrangian multi-sections with the same mirror object are in fact lifted-Hamiltonian isotopic.

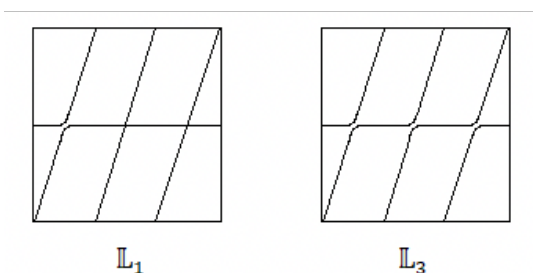


Figure 3. Two lifted-Hamiltonian isotopic Lagrangian multi-section in the 2-torus

Mirror symmetry is one of the most striking phenomena in the entire history of mathematics. It builds a bridge between two completely different geometries on two completely different spaces. Later, mathematicians and physicists have applied the idea of mirror symmetry to build more connections in other fields of mathematics. Our work has been published in *Transaction in American Mathematical Society* in May 2019.

## References

1. Kwokwai Chan and **Yat-Hin Suen**, *SYZ transforms for immersed Lagrangian multisections*, Trans. Amer. Math. Soc. 372 (2019), no. 8, 5747–5780, MR 4014293.

## Interviews

Yunhyung Cho

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Gabriel C. Drummond-Cole

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Anna Cepek

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Hee-Joong Chung

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**Yunhyung Cho***Research member from October 2015 to February 2017*

How is your life / work after CGP?

I have been working at Sungkyunkwan University (SKKU) Department of Mathematics Education located in Seoul since 2017. As the students in my department mostly want to be math teachers, my job is mainly focused on teaching. One of the great benefits of SKKU is the location; it is quite close to KIAS and so I often attend seminars or meet my colleagues at KIAS and have discussion. This really helps me to continue my research together with my collaborators.

What made you decide to be a mathematician?

I believe that every law of nature is governed by “something” (maybe called a truth) unchangeable and mathematics is a field of study understanding the “something”. There are many unanswered mathematical phenomena of the universe and mathematicians explore them and discovered solutions. *Mathematics is a history of nature in that sense and I believe that mathematics answers for everything.* This explains why mathematics is attractive enough, isn't it?

What is your current math-related interest?  
Please tell us about your research.

I mainly study some phenomenon (called a mutation or a wall-crossing) which appears in various contexts of mirror symmetry. On algebraic side, there is a notion of a “mutation” of Laurent polynomials; roughly speaking, two Laurent polynomials are said to be “mutation-equivalent” if they give the same period integral and one can define “mutation-equivalence” of Laurent polynomials in terms of axioms. The notion of a mutation can be generalized to a lattice polytope such that two mutation-equivalent Laurent polynomials induce the mutation-equivalent Newton polytopes of the Laurent polynomials.

On the other hand, there is a notion of a mutation on geometric side, called a “mutation (or a wall-crossing) of Lagrangian tori” of a symplectic manifold. In a suitable situation, we can assign a Laurent polynomial (called a Fukaya-Oh-Ohta-Ono's disc potential) to each Lagrangian torus and it is expected that the mutation structure of Lagrangians is compatible with that of Laurent polynomials.

I am currently trying to prove the compatibility between the mutation structures of Laurent polynomials and Lagrangian tori using the tropical geometry and toric degenerations. I believe that we can find a very clear explanation of the phenomenon in view of cluster algebras.

What are you interested in recently?  
Please share something about you.

From this fall semester, I am teaching “mathematics behind ML (machine learning)”. Honestly, I was not familiar with machine learning even 6 months ago. During preparation of ML class, I figured out that the goal of a machine

learning is just minimizing a function (called the cost function) and so the basic idea of ML comes from Calculus and numerical analysis.

I found one interesting part, which is that the current version of ML uses some simple setup to build up an algorithm (such as a sigmoid function or a ReLU function) and many experimental results show that this setup works well, but nobody knows the reason why it behaves nicely. It will be interesting if we can translate ML problem in terms of our math terminologies.



We would like to hear about your dream or future plans.  
Do you have a role model or a philosophy of life? :)

For my math life, I have a plan to study various kinds of mathematics. I have mainly studied topology and geometry. Nowadays I feel that **studying a special math topic is like studying one language**. My recent topic is closely related to other areas such as algebraic geometry, representation theory, and combinatorics. I recognize that I can make much more progress on this topic if I could understand other related areas. Eventually I would like to discuss and study with various other math people.

My role model is anybody who can enjoy his/her own life. The word “enjoy” has lots of meaning for me; enjoy my career, enjoy life with my lovely family, enjoy my own time. Recently I figure out that I spend most of my time at work and I am losing my small happiness. Hopefully, I would like to spend my time with my family and friends as much as possible.

**Gabriel C. Drummond-Cole***Research member from September 2013 to July 2019*

How is your life / work after CGP?

I'm living in California, in Silicon Valley. I spent a few months preparing to apply to industry jobs and now I am working at a big tech company as a data scientist. Because of COVID-19 we're all working remotely so I sit around my apartment most of the time working on things and having videoconferences. There are interesting long term questions to think about and always something new to do.

What are you interested in recently?

I still like board games and play them online with friends from time to time. Usually I go on a hike every week. The area where I live has rugged hills, the ocean, redwood forests, and the San Francisco Bay, so there are many types of hike to try. I've been reading a lot too; I recently finished the second volume of the Gulag Archipelago. I know that Solzhenitsyn isn't Sasha's favorite, but I found it a compelling and heartbreaking read.



How was your life in Korea comparing to one in the US?

Please share your story.

I miss a lot of things about Pohang. I liked Yugang, the trails on the hills between Yugang and Jamyong and the University were a great place to walk and here I can't just walk to a trail like that. I live near a Korean neighborhood so I can eat jjol-myeon any time I want, but I haven't found satisfying kong guksu or memil guksu yet. But on the other hand, there are a lot more vegetarian options at the restaurants where I live now. Another thing I like better where I am now is the weather; I don't miss the humid Pohang summers at all!



Is there anything you want to tell younger mathematicians?

Everyone has a different journey so it's hard to know what to say. If I were talking to a younger version of myself I'd say *"follow the kind of problems you like and don't worry too much about whether anyone else thinks they're interesting"* and also *"stick with smaller projects, you'll have more fun with them"* but I guess both of those would be bad advice for some people.

We would like to hear about your dream or future plans.  
Do you have a role model or a philosophy of life? :)

With 2020 being as turbulent as it has been I've just been taking things day by day. I'm satisfied with my life for now, so I think I'll keep on my current path as long as it feels okay and then maybe try something else. I've never had a coherent philosophy of life so I don't know about that, but two historical figures I look to for inspiration are Emma Goldman and James Baldwin.

## Anna Ceppek

Research member since August 2019

How is your life in CGP / Pohang / Korea?

I have grown a lot since moving to Korea, both mathematically and personally. CGP is a positive work environment, giving me the freedom to pursue the math that I love. Everyone at CGP, from the director to the researchers to the secretaries, is so kind.

I really enjoy certain aspects of Pohang and South Korea, such as all the woods and mountains and all of the hiking trails, the sea and the river, the persimmons, and the bamboo.

Honestly, being a foreigner in Pohang has been hard, especially coming from a country that has such a different culture. But due to the kindness of a few individuals around me, I am finding myself more and more at home here. I especially enjoy having a weekly dinner with Eunju and her family; they have welcomed me into their home and have adopted me as their 'Auntie Anna'. Her daughters are raw energy and love; they bring so much joy to my life.

What made you decide to be a mathematician?

**Math is black and white**, unlike real life. Problems can be solved definitively and are not subject to personal perspective. In that way, math is quite a refreshing break from the 'real world'. Perhaps surprisingly, it requires a lot of creativity. In doing mathematics, I often feel that I am exploring a very large landscape, and like Lewis and Clark exploring the western United States, I encounter many surprises, many problems, and have many triumphs.

In college, I continued to study math because it was the most enjoyable subject for me, and the most challenging - that was the beginning. And then I just kept studying it until I found myself to be a professional mathematician. I have also had much encouragement along the way, from mentors and friends, peers, and family. At the end of the day, I am a mathematician because I really enjoy spending my time thinking about the math that I do.



with Eunju's daughters

What is your current math-related interest?  
Please tell us about your research.

I study algebraic topology, which tries to apply algebra to understand certain aspects of spaces, such how many holes a space has and what dimension the holes are. In particular, I study spaces using finite subsets of points in the spaces, which you could imagine a little dots floating around the space. For the experts, I study configuration spaces of finite subsets of manifolds using higher category theory. I'm particularly interested in configuration spaces of Euclidean space and the circle.

What are you interested in recently?  
Please share something about you.

My favorite hobby is running in the mountains. Being surrounded by nature is stimulating for my mind and emotions, and the actual act of running in the mountains - weaving my way along a foot trail over rocks and roots, under trees, and through bushes, scrambling up and flying down inclines, jumping across streams, hopping from rock to rock, and navigating through the wilderness - is physically and emotionally challenging, and absolutely exhilarating. Besides running in the woods in and near Pohang, I've visited many mountains in Korea, some of my favorites being Seonginbong on Ulleung-do, Jirisan, Juwangsansan, Unjesan and Seoraksan.



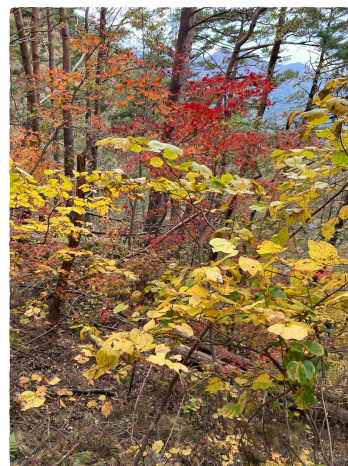


We would like to hear about your dream or future plans.  
Do you have a role model or a philosophy of life? :)

For the near future, I would like to continue to study and research math. My long term future dreams involve teaching math and having a farm with a group of people, owning cats and dogs, chickens, and goats, having a vegetable garden and cultivating a food forest over the span of 20 years.

I have a few role models. One is my father, who is my favorite person in all of the world. He is kind and gentle, loving and wise. He has taught me who God is and what Love is. Knowing both of these things has brought me fullness of life.

Another role model is my PhD advisor, David Ayala, who has taught me a perspective in life that fuels my each and every step, both in math and personally. From him I learned to value my time, to avoid making decisions based on fear, and to 'just focus on the math' rather than defining my abilities relative to those who are around me.



**Hee-Joong Chung***Research member since September 2020*

How is your life in CGP / Pohang / Korea?

It is overall good. I have visited CGP several times in the past because my collaborators were at CGP and POSTECH. So the atmosphere is quite familiar to me, though nowadays many things have changed due to COVID-19. I hope that the situation gets better soon so that CGP runs very actively as before.

Before I came back to Korea, I was at Yau Mathematical Sciences Center, Tsinghua University in Beijing. I visited Korea around the middle of this January because the winter break began, and fortunately it was just before COVID-19 became very serious in China. Since then, I had stayed in Seoul until I joined CGP in September. It is nice that I am living in my home country, especially that I can visit my home in Seoul more often. Though I have visited Pohang several times, I haven't explored the city. After COVID-19 is gone, I hope to visit some nice places in Pohang.

What made you decide to be a physicist?

Since I was an elementary school student, somehow I have been interested in science and in particular I felt that physics looked cool. I read some books on physics for the public, for example, about the theory of relativity, quantum mechanics, and elementary particles. I felt that things like time dilation or quantum tunneling were strange and interesting when I was a high school student. So I chose physics as my major. When I was an undergraduate student, I was inclined to theoretical physics and I thought that it was interesting. Accordingly, I thought that it would be nice to continue studying it and to be a physicist.

I think the attractive point of physics is that it is about pushing the boundary of understanding and knowledge on Nature. In particular, I think quantum field theory and string/M-theory is very attractive in that they contain a lot of beautiful ideas. In addition, from the viewpoint of mathematical physics, they have very rich structures.

What is your current physics-related interest?  
Please tell us about your research.

Recently, I have been interested in the so-called homological block, which is a new topological invariant of 3-manifolds. It is labelled by  $\text{Spin}^c$  structure of the 3-manifold and can be regarded as a building block of an analytically continued Chern-Simons partition function. Some interesting properties have been discussed in the context of modularity, Witten-Reshetikhin-Turaev invariants, knot invariants, resurgence, log-CFT, etc. I am working on several physical and mathematical aspects of it. In general, I am interested in formal aspects of quantum field theory and string/M-theory, and mathematical physics.

What are you interested in recently?  
Please share something about you.

My interest is to spend time with my baby boy. When I was in China, I visited my home in Korea around once in 3 - 4 weeks and spent several days or visited about a month during summer or winter breaks. So I didn't spend much time with my son while I was in China. After arriving at Pohang, I visit my home every weekend, so I can visit my family more often, which is nicer.

Before I obtained my Ph.D., I played piano from time to time. After that, piano was not around me usually, so I barely played piano for a while. Meanwhile, since a digital piano is at CGP Hall, I might play it from time to time. I also like to listen to various genres of music. I used to go to concert halls to listen to classical music.



We would like to hear about your dream or future plans.  
Do you have a role model or a philosophy of life? :)

As a researcher, I would like to make great discoveries. Also, I would like to have a permanent job soon. I don't have particular role models usually, because there are many people from whom I learn various lessons. But if I have to choose, academically my role model would be my Ph.D. advisors, Sergei Gukov and John H. Schwarz. I used not to set particularly a philosophy of life, but I like a phrase, just do it.

# Senior Researcher Positions\*

IN GEOMETRY AND MATHEMATICAL PHYSICS

## I LOCATION

POSTECH Campus in Pohang, Gyeongbuk

## I CURRENT AREAS OF RESEARCH

Symplectic geometry, geometric topology, dynamical systems, algebraic geometry, number theory, and mathematical aspects of quantum field and string theory

IBS-CGP opens **senior researcher positions** every year.

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Step 1) Please apply online at <http://cgp.ibs.re.kr/jobopenings/apply> with a cover letter, CV including the list of publications, research statement and consent form for collection and use of personal information.

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