Low-dimensional Topology and Geometric Group Theory

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Abstract.

We will discuss various perspective to study low-dimensional spaces, namely 2 or 3-dimensional manifolds.

- (Lecture 1) We will provide a basic introduction to hyperbolic geometry (mostly in dimension 2). Here the hyperbolic space means a Riemannian metric with constant negative curvature -1. After that, we will introducer the notion of large-scale geometry (or coarse geometry). Two metric spaces are said to be coarsely equivalent (or quasi-isometric) if they look about the same when we see them from a very very far away. This flexible notion of "geometry" will allow us to generalize the notion of hyperbolicity so that we do not have to worry too much about the classical notion of the curvature.
- (Lecture 2) We will introduce the notion of mapping class groups and some metric spaces on which mapping class groups act. Here a mapping class group is the group of homeomorphisms from a space to itself up to homotopy (i.e., a continuous deformation). The elements of mapping class groups for surfaces have been classified by Thurston in the geometric topology perspective which was reproved by Bers in more analytic terms.
- (Lecture 3) We will discuss various recent researches using the notions introduced in two previous lectures.

References.

- Lecture note by prof. Caroline Series.
  http://homepages.warwick.ac.uk/ masbb/Papers/MA448.pdf
- Farb, Benson, and Dan Margalit. A primer on mapping class groups (pms-49). Princeton University Press, 2011.
- Lh, Clara. Geometric group theory. Springer International Publish- ing AG, 2017.

Working Problems.

Q1. Let G be a finitely generated group, and A, B be two finite generating set of G. Show that $Cay(G, A)$ and $Cay(G, B)$ are quasi-isometric.

Q2. Show that a quasi-isometry always admits a quasi-inverse. Using this, show that qausi-isometry among metric spaces is an equivalence relation.

Q3. What are the mapping class groups of S^2, D^2, $S^1 \times [0, 1]$, T^2?

Q4. How would you describe the mapping class groups of punctured disks (a closed 2-dimensional disk with finitely many points removed from the interior)?

Q5. Show the latern relation in the mapping class group.

Q6. Show that the curve complex of T^2 is isomorphic to the Farey graph.