Legendrians, Cluster algebras, and Mirror symmetry

January 4–15, 2021

Program and Abstracts

Legendrians, Cluster algebras, and Mirror symmetry

School January 4–8, 2021 | **Conference** January 11–15, 2021

Online (Zoom)

School Lecturers

Cheol-Hyun Cho (Seoul National University)
Yunhyung Cho (Sungkyunkwan University)
Honghao Gao (Michigan State University)
Tatsuki Kuwagaki (Osaka University)
Linhui Shen (Michigan State University)
Daping Weng (Michigan State University)

Conference Speakers

Byung Hee An (Kyungpook National University)
Roger Casals (University of California, Davis)
Naoki Fujita (The University of Tokyo)
Benjamin Gammage (Harvard University)
Honghao Gao (Michigan State University)
Hyun Kyu Kim (Ewha Womans University)
Tatsuki Kuwagaki (Osaka University)
Thomas Lam (University of Michigan)
Sangwook Lee (Soongsil University)
Lenhard L. Ng (Duke University)
Linhui Shen (Michigan State University)
Yat-Hin Suen (IBS-CGP)
Daping Weng (Michigan State University)
Eric Zaslow (Northwestern University)

Organizers

Byung Hee An (Kyungpook National University)
Youngjin Bae (Incheon National University)
Eunjeong Lee (IBS-CGP)
Yong-Geun Oh (IBS-CGP)

Contact

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NRF 한국연구재단 National Research Foundation of Korea

Legendrians, Cluster algebras, and Mirror symmetry

January 4–15, 2021

Organizers

- Byunghee An (Kyungpook National University)
- Youngjin Bae (Incheon National University)
- Eunjeong Lee (IBS Center for Geometry and Physics)
- Yong-Geun Oh (IBS Center for Geometry and Physics)

Sponsored by

- Center for Geometry and Physics, Institute for Basic Science
- National Research Foundation of Korea
  - Grant No. : 2020R1A2C1A01003201
    (led by Byung Hee An and Youngjin Bae)
- Department of Mathematics, Incheon National University
- Department of Mathematics Education, Kyungpook National University

Program: School

January 4 (Monday) KST (UTC+9)

08:50–09:00	Greetings	
09:00–10:00	**Sheaves in Contact Topology I** **Legendrian knots and sheaves** Honghao Gao (Michigan State University)	p.9
10:00–11:00	**Sheaves in Contact Topology II** **Invariance** Honghao Gao (Michigan State University)	p.9
13:00–14:00	**Homological mirror symmetry via Lagrangian Floer theory I** **A-infinity category, HMS and localized mirror functor** Cheol-Hyun Cho (Seoul National University)	p.11
14:00–15:00	**Homological mirror symmetry via Lagrangian Floer theory II** **Monotone Floer theory and its HMS** Cheol-Hyun Cho (Seoul National University)	p.11
15:30–16:30	**Mutations and toric degenerations I** **Fano toric varieties and potentials** Yunhyung Cho (Sungkyunkwan University)	p.13
16:30–17:30	**Mutations and toric degenerations II** **Toric degenerations, examples and construction** Yunhyung Cho (Sungkyunkwan University)	p.13

January 5 (Tuesday) KST (UTC+9)

09:00–10:00	**Sheaves in Contact Topology III** **Moduli space of sheaves** Honghao Gao (Michigan State University)	p.9
10:00–11:00	**Sheaves in Contact Topology IV** **Lagrangian fillings** Honghao Gao (Michigan State University)	p.9
13:00–14:00	**Homological mirror symmetry via Lagrangian Floer theory III** **Fukaya category of surfaces and its HMS** Cheol-Hyun Cho (Seoul National University)	p.11
14:00–15:00	**Homological mirror symmetry via Lagrangian Floer theory IV** **Singularities and its HMS** Cheol-Hyun Cho (Seoul National University)	p.11
15:30–16:30	**Mutations and toric degenerations III** **Mutations of potentials** Yunhyung Cho (Sungkyunkwan University)	p.13
16:30–17:30	**Mutations and toric degenerations IV** **Examples: flag variety** Yunhyung Cho (Sungkyunkwan University)	p.13

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09:00–10:00	**An introduction to cluster algebras I** **Cluster algebras of rank 2: positive Laurent phenomenon and greedy bases** Linhui Shen (Michigan State University)	p.15
10:00–11:00	**An introduction to cluster algebras II** **Cluster algebras and finite type classifications** Linhui Shen (Michigan State University)	p.15

January 7 (Thursday) KST (UTC+9)

09:00–10:00	**An introduction to cluster algebras III** **Poisson geometry and quantization** Linhui Shen (Michigan State University)	p.15
10:00–11:00	**Examples of cluster varieties from plabic graphs I** **Gr$(2,n)$ and $M(0,n)$** Daping Weng (Michigan State University)	p.17
11:00–12:00	**Examples of cluster varieties from plabic graphs II** **Plabic graphs and Gr(k,n)** Daping Weng (Michigan State University)	p.17
14:00–15:00	**Symplectic geometry in algebraic analysis I** **Sheaf quantization: basic ideas and examples** Tatsuki Kuwagaki (Osaka University)	p.19
15:00–16:00	**Symplectic geometry in algebraic analysis II** **Sheaf quantization: continued** Tatsuki Kuwagaki (Osaka University)	p.19

January 8 (Friday) KST (UTC+9)

09:00–10:00	**An introduction to cluster algebras IV** **Categorification and Donaldson–Thomas theory** Linhui Shen (Michigan State University)	p.15
10:00–11:00	**Examples of cluster varieties from plabic graphs III** **Double Bruhat cells of SL$_n$** Daping Weng (Michigan State University)	p.17
11:00–12:00	**Examples of cluster varieties from plabic graphs IV** **Double Bott–Samelson cells of SL$_n$ and positive braid closures** Daping Weng (Michigan State University)	p.17
14:00–15:00	**Symplectic geometry in algebraic analysis III** **Exact WKB analysis: basics** Tatsuki Kuwagaki (Osaka University)	p.19
15:00–16:00	**Symplectic geometry in algebraic analysis IV** **Exact WKB analysis: cluster algebra and local systems** Tatsuki Kuwagaki (Osaka University)	p.19

Program: Conference

January 11 (Monday) KST (UTC+9)

08:50–09:00	Greetings	
09:00–10:00	**Legendrian knots and their Lagrangian fillings: A conspectus on recent developments** Roger Casals (University of California, Davis)	p.25
10:00–11:00	**Infinitely many fillings through augmentations** Lenhard L. Ng (Duke University)	p.25
11:00–12:00	**Infinitely many fillings through sheaves** Honghao Gao (Michigan State University)	p.25

January 12 (Tuesday) KST (UTC+9)

09:00–10:00	**Lagrangian fillings of Legendrian links of finite type** Byung Hee An (Kyungpook National University)	p.26
10:00–11:00	**Quantum geometry of moduli spaces of local systems** Linhui Shen (Michigan State University)	p.26
11:00–12:00	**Symplectic Structure on Augmentation Varieties** Daping Weng (Michigan State University)	p.26

January 13 (Wednesday) KST (UTC+9)

09:00–10:00	**Positroid varieties and q,t-Catalan numbers** Thomas Lam (University of Michigan)	p.27
10:00–11:00	**Newton–Okounkov bodies arising from cluster structures and mutations on polytopes** Naoki Fujita (The University of Tokyo)	p.27
11:00–12:00	**SL_3-laminations as bases for PGL_3 cluster varieties for surfaces** Hyun Kyu Kim (Ewha Womans University)	p.27

January 14 (Thursday) KST (UTC+9)

09:00–10:00	**Mirror symmetry through perverse schobers** Benjamin Gammage (Havard University)	p.28
10:00–11:00	**Tropical Lagrangian multi-sections and smoothing of locally free sheaves on log Calabi–Yau surfaces** Yat-Hin Suen (IBS Center for Geometry and Physics)	p.28
11:00–12:00	**Orbifold Jacobian algebras and generalized Kodaira–Spencer maps** Sangwook Lee (Soongsil University)	p.28

January 15 (Friday) KST (UTC+9)

09:00–10:00	**Cluster coordinates from sheaf quantization of spectral curve** Tatsuki Kuwagaki (Osaka University)	p.29
10:00–11:00	**Dimers and Mirror Moduli** Eric Zaslow (Northwestern University)	p.29

School

LECTURE SERIES 1

Sheaves in Contact Topology

by Honghao Gao (Michigan State University)

Abstract Microlocal sheaf theory was introduced by Kashiwara–Schapira around 80s. With the notion of micro-support, one can use sheaves on smooth manifolds to access the geometry of their cotangent bundles. In recent years, microlocal sheaf theory entered contact and symplectic topology, and has been used to solve open problems. In this lecture series, we will introduce microlocal sheaf theory in the context of low-dimensional contact topology, and supply the audience with background for its applications such as producing non-classical invariants for Legendrian knots and distinguishing exact Lagrangian fillings.

Lecture 1. Legendrian knots and sheaves

January 4 (Monday), 09:00–10:00 KST (UTC+9)
16:00–17:00 PDT (-1day) | 19:00–20:00 EDT (-1day) | 13:00–14:00 NZST

Basics of Legendrian knots, sheaves and microsupport, local conditions at arcs, cusps, crossings.

Lecture 2. Invariance

January 4 (Monday), 10:00–11:00 KST (UTC+9)
17:00–18:00 PDT (-1day) | 20:00–21:00 EDT (-1day) | 14:00–15:00 NZST

Category of sheaves, non-classical invariants for Legendrian submanifolds (theorem by Guillermou–Kashiwara–Schapira), combinatorial verification under Reidemeister moves.

Lecture 3. Moduli space of sheaves

January 5 (Tuesday), 09:00–10:00 KST (UTC+9)
16:00–17:00 PDT (-1day) | 19:00–20:00 EDT (-1day) | 13:00–14:00 NZST

Moduli space of sheaves for elementary tangles, microlocal rank 1 sheaves, positive braid Legendrian knots, flags and Bott–Samelson cells.

Lecture 4. Lagrangian fillings

January 5 (Tuesday), 10:00–11:00 KST (UTC+9)
17:00–18:00 PDT (-1day) | 20:00–21:00 EDT (-1day) | 14:00–15:00 NZST

Singularities of Legendrian fronts, exact Lagrangian fillings and Legendrian weaves, sheaf quantization of Lagrangian fillings.

LECTURE SERIES 2

Homological mirror symmetry via Lagrangian Floer theory

by Cheol-Hyun Cho (Seoul National University)

Abstract A version of homological mirror symmetry (HMS) conjecture relates the Fukaya category of a symplectic manifold and matrix factorization category of a mirror Landau–Ginzburg model. In this introductory lecture series, we illustrate geometric ideas behind such correspondences from a biased point of view of the theory of localized mirror functor in Lagrangian Floer theory.

Lecture 1. A-infinity category, HMS and localized mirror functor

January 4 (Monday), 13:00–14:00 KST (UTC+9)
20:00–21:00 PDT (-1day) | 23:00–24:00 EDT (-1day) | 17:00–18:00 NZST

Lecture 2. Monotone Floer theory and its HMS

January 4 (Monday), 14:00–15:00 KST (UTC+9)
21:00–22:00 PDT (-1day) | 00:00–01:00 EDT | 18:00–19:00 NZST

Lecture 3. Fukaya category of surfaces and its HMS

January 5 (Tuesday), 13:00–14:00 KST (UTC+9)
20:00–21:00 PDT (-1day) | 23:00–24:00 EDT (-1day) | 17:00–18:00 NZST

Lecture 4. Singularities and its HMS

January 5 (Tuesday), 14:00–15:00 KST (UTC+9)
21:00–22:00 PDT (-1day) | 00:00–01:00 EDT | 18:00–19:00 NZST

LECTURE SERIES 3

Mutations and toric degenerations

by Yunhyung Cho (Sungkyunkwan University)

Abstract The aim of this lecture is to understand a relation between the wall crossing phenomenon of Lagrangians and the mutations in cluster theory via toric degenerations.

Lecture 1. Fano toric varieties and potentials

January 4 (Monday), 15:30–16:30 KST (UTC+9)
22:30–23:30 PDT (-1day) | 01:30–02:30 EDT | 19:30–20:30 NZST

- A brief introduction to toric varieties
- Potential functions of smooth Fano toric varieties

Lecture 2. Toric degenerations, examples and construction

January 4 (Monday), 16:30–17:30 KST (UTC+9)
23:30–24:30 PDT (-1day) | 02:30–03:30 EDT | 20:30–21:30 NZST

- Toric degenerations: definitions and examples
- Construction of toric degenerations
- Potential functions via toric degenerations

Lecture 3. Mutations of potentials

January 5 (Tuesday), 15:30–16:30 KST (UTC+9)
22:30–23:30 PDT (-1day) | 01:30–02:30 EDT | 19:30–20:30 NZST

- Mutations of Laurent polynomials, polytopes, and Lagrangian tori

Lecture 4. Examples: flag variety

January 4 (Monday), 16:30–17:30 KST (UTC+9)
23:30–24:30 PDT (-1day) | 02:30–03:30 EDT | 20:30–21:30 NZST

- Toric degenerations of flag varieties
- Cluster structures of G/B and potential functions

LECTURE SERIES 4

An introduction to cluster algebras

by Linhui Shen (Michigan State University)

Abstract Cluster algebras are commutative algebras equipped with remarkable combinatorial structures. Since its inception in 2000, the theory of cluster algebras has found numerous exciting applications in mathematics and physics. This series of lectures aim to provide an accessible introduction to cluster algebras for a general mathematical audience. In particular, we will investigate the following topics.

Lecture 1. Cluster algebras of rank 2: positive Laurent phenomenon and greedy bases

January 6 (Wednesday), 09:00–10:00 KST (UTC+9)
16:00–17:00 PDT (-1day) | 19:00–20:00 EDT (-1day) | 13:00–14:00 NZST

This lecture will focus on cluster algebras of rank 2. Using elementary combinatorial tools, we will prove the positive Laurent Phenomenon and construct greedy bases for cluster algebras of rank 2.

Lecture 2. Cluster algebras and finite type classifications

January 6 (Wednesday), 10:00–11:00 KST (UTC+9)
17:00–18:00 PDT (-1day) | 20:00–21:00 EDT (-1day) | 14:00–15:00 NZST

We begin with a rigorous definition of cluster algebras in terms of quiver mutations. We present a classification of cluster algebras of finite types by ADE quivers and explain their connections to generalized associahedra.

Lecture 3. Poisson geometry and quantization

January 7 (Thursday), 09:00–10:00 KST (UTC+9)
16:00–17:00 PDT (-1day) | 19:00–20:00 EDT (-1day) | 13:00–14:00 NZST

Cluster varieties carry intrinsic Poisson structures. We present a quantization of cluster varieties and explore their connections with the theory of quantum groups.

Lecture 4. Categorification and Donaldson–Thomas theory

January 7 (Friday), 09:00–10:00 KST (UTC+9)
16:00–17:00 PDT (-1day) | 19:00–20:00 EDT (-1day) | 13:00–14:00 NZST

Every cluster variety can be categorized and gives rise to a 3d Calabi–Yau category with a generic stability condition. In this lecture, we will investigate their connections to the motivic Donaldson–Thomas theory.

LECTURE SERIES 5

Examples of cluster varieties from plabic graphs

by Daping Weng (Michigan State University)

Abstract Cluster varieties were introduced by Fock and Goncharov as geometric counterparts of Fomin and Zelevinsky's cluster algebras. Simply speaking, cluster varieties are algebraic varieties with an atlas of torus charts, whose transition maps are captured by certain combinatorial process called cluster mutations. Many interesting geometric objects turn out to be examples of cluster varieties, and one can then use cluster theoretical techniques to study these geometric objects. In this lecture series, we will discuss various examples of cluster varieties whose combinatorics can be captured by plabic graphs, including Grassmannians and double Bruhat/Bott–Samelson cells of SL_n. This lecture series will be complementary to Linhui Shen's lecture series on cluster theory.

Lecture 1. $\mathrm{Gr}(2,n)$ **and** $M(0,n)$

January 7 (Thursday), 10:00–11:00 KST (UTC+9)
17:00–18:00 PDT (-1day) | 20:00–21:00 EDT (-1day) | 14:00–15:00 NZST

We discuss the cluster structures on Grassmannian $\mathrm{Gr}(2,n)$ and on the moduli space of n points in $\mathbb{P}^1$. These are examples of cluster varieties of Dynkin A_{n-3} mutation type and their combinatorics are captured by triangulations of an n-gon.

Lecture 2. Plabic graphs and $\mathrm{Gr}(k,n)$

January 7 (Thursday), 11:00–12:00 KST (UTC+9)
18:00–19:00 PDT (-1day) | 21:00–22:00 EDT (-1day) | 15:00–16:00 NZST

We introduce plabic (planar bicolor) graphs and use them to describe the cluster structures on Grassmannian $\mathrm{Gr}(k,n)$ and on the moduli space of n points on $\mathbb{P}^{k-1}$.

Lecture 3. Double Bruhat cells of SL_n

January 8 (Friday), 10:00–11:00 KST (UTC+9)
17:00–18:00 PDT (-1day) | 20:00–21:00 EDT (-1day) | 14:00–15:00 NZST

We introduce double Bruhat cells of a semisimple Lie group and discuss the cluster structures on double Bruhat cells of SL_n in terms of plabic graphs.

Lecture 4. Double Bott–Samelson cells of SL_n **and positive braid closures**

January 8 (Friday), 11:00–12:00 KST (UTC+9)
18:00–19:00 PDT (-1day) | 21:00–22:00 EDT (-1day) | 15:00–16:00 NZST

We introduce double Bott–Samelson cells of SL_n as a generalization of double Bruhat cells. We will describe their cluster structures and the connection to positive braid closures.

LECTURE SERIES 6

Symplectic geometry in algebraic analysis

by Tatsuki Kuwagaki (Osaka University)

Abstract In these lectures, I will explain two ideas in algebraic analysis: sheaf quantization and exact WKB analysis, with emphasis on relations to symplectic geometry. The ideas presented in the lectures will be used in my talk in the workshop.

Lecture 1. Sheaf quantization: basic ideas and examples

January 7 (Thursday), 14:00–15:00 KST (UTC+9)
21:00–22:00 PDT (-1day) | 00:00–01:00 EDT | 18:00–19:00 NZST

Lecture 2. Sheaf quantization: continued

January 7 (Thursday), 15:00–16:00 KST (UTC+9)
22:00–23:00 PDT (-1day) | 01:00–02:00 EDT | 19:00–20:00 NZST

Lecture 3. Exact WKB analysis: basics

January 8 (Friday), 14:00–15:00 KST (UTC+9)
21:00–22:00 PDT (-1day) | 00:00–01:00 EDT | 18:00–19:00 NZST

Lecture 4. Exact WKB analysis: cluster algebra and local systems

January 8 (Friday), 15:00–16:00 KST (UTC+9)
22:00–23:00 PDT (-1day) | 01:00–02:00 EDT | 19:00–20:00 NZST

Timetable: School

	08	09		10		11		13		14		15		16		17	
	50	00	30	00	30	00	30	00	30	00	30	00	30	00	30	00	30
Jan 4 (Mon)	Greetings	Gao I		Gao II				C. Cho I		C. Cho II			Y. Cho I		Y. Cho II		
Jan 5 (Tue)		Gao III		Gao IV				C. Cho III		C. Cho IV			Y. Cho III		Y. Cho IV		
Jan 6 (Wed)		Shen I		Shen II													
Jan 7 (Thu)		Shen III		Weng I		Weng II				Kuwagaki I		Kuwagaki II					
Jan 8 (Fri)		Shen IV		Weng III		Weng IV				Kuwagaki III		Kuwagaki IV					

Local time

KST (UTC+9)	08	09	10	11	13	14	15	16	17
PDT (UTC-8)	15 (-1day)	16 (-1day)	17 (-1day)	18 (-1day)	20 (-1day)	21 (-1day)	22 (-1day)	23 (-1day)	00
EDT (UTC-5)	18 (-1day)	19 (-1day)	20 (-1day)	21 (-1day)	23 (-1day)	00	01	02	03
NZST (UTC+13)	12	13	14	15	17	18	19	20	21

Conference

January 11

Legendrian knots and their Lagrangian fillings: A conspectus on recent developments

by Roger Casals (University of California, Davis)

09:00–09:50 KST (UTC+9)

16:00–16:50 PDT (-1day) | 19:00–19:50 EDT (-1day) | 13:00–13:50 NZST

Abstract In this talk, I will survey some of the recent developments in the study of Lagrangian fillings of Legendrian knots. First, I will introduce and motivate the leading questions. Then, we will discuss the current methods and techniques available to tackle them. Finally, I will suggest some open problems that now seem at reach, along with some strategies to approach them.

Infinitely many fillings through augmentations

by Lenhard L. Ng (Duke University)

10:00–10:50 KST (UTC+9)

17:00–17:50 PDT (-1day) | 20:00–20:50 EDT (-1day) | 14:00–14:50 NZST

Abstract In 2020, a few groups of people proved that certain Legendrian links in $\mathbb{R}^3$ have infinitely many exact Lagrangian fillings that are distinct under Hamiltonian isotopy. These groups (Casals–Gao, Gao–Shen–Weng, Casals–Zaslow) used a variety of approaches involving microlocal sheaf theory and cluster structures. I'll describe a different, Floer-theoretic approach to the same sort of result, using integer-valued augmentations of Legendrian contact homology, and I'll discuss some examples that are amenable to the Floer approach but not (yet?) the other approaches. This is joint work with Roger Casals.

Infinitely many fillings through sheaves

by Honghao Gao (Michigan State University)

11:00–11:50 KST (UTC+9)

18:00–18:50 PDT (-1day) | 21:00–21:50 EDT (-1day) | 15:00–15:50 NZST

Abstract This talk will complement other talks in the day and present concrete examples. Specifically, I will construct infinitely many Lagrangian fillings for the Legendrian torus link $(3, 6)$, and explain how to distinguish them using sheaves and cluster algebras. Time permitting, I will discuss other torus links (joint work with R. Casals) and positive braid links (joint work with L. Shen and D. Weng).

January 12

Lagrangian fillings of Legendrian links of finite type

by Byung Hee An (Kyungpook National University)

09:00–09:50 KST (UTC+9)

16:00–16:50 PDT (-1day) | 19:00–19:50 EDT (-1day) | 13:00–13:50 NZST

Abstract In this talk, we will focus on Legendrian links admitting cluster structures of finite type (via N-graph ways) and prove that those Legendrian links of type ADE have embedded exact Lagrangian fillings as many as the number of seeds in their cluster structures.

Furthermore, we will describe the cluster structures of $BCFG$-type among Lagrangian fillings of ADE-type Legendrian links, which have certain partial symmetries.

Quantum geometry of moduli spaces of local systems

by Linhui Shen (Michigan State University)

10:00–10:50 KST (UTC+9)

17:00–17:50 PDT (-1day) | 20:00–20:50 EDT (-1day) | 14:00–14:50 NZST

Abstract Let G be a split semi-simple algebraic group over $\mathbb{Q}$. We introduce a natural cluster structure on moduli spaces of G-local systems over surfaces with marked points. As a consequence, the moduli spaces of G-local systems admit natural Poisson structures, and can be further quantized. We will study the principal series representations of such quantum spaces. It will recover many classical topics, such as the q-deformed Toda systems, quantum groups, and the modular functor conjecture for such representations. This talk will mainly be based on joint work with A.B. Goncharov.

Symplectic Structure on Augmentation Varieties

by Daping Weng (Michigan State University)

11:00–11:50 KST (UTC+9)

18:00–18:50 PDT (-1day) | 21:00–21:50 EDT (-1day) | 15:00–15:50 NZST

Abstract In a recent joint project with H. Gao and L. Shen, we introduce a cluster K2 structure on the augmentation variety of the Chekanov–Eliashberg dga for the rainbow closure of any positive braid with marked point decorations. This cluster K2 structure defines a holomorphic presymplectic structure on the complex augmentation variety. Using a result of Goncharov and Kenyon on surface bipartite graphs, we prove that this holomorphic presymplectic structure becomes symplectic after we reduce the number of marked points to a single marked per link component.

January 13

Positroid varieties and q,t-Catalan numbers

by Thomas Lam (University of Michigan)

09:00–09:50 KST (UTC+9)

16:00–16:50 PDT (-1day) | 19:00–19:50 EDT (-1day) | 13:00–13:50 NZST

Abstract Positroid varieties are subvarieties of the Grassmannian defined as intersections of rotations of Schubert varieties in my work with Knutson and Speyer. They also appear in the work of Shende–Treumann–Williams–Zaslow as moduli spaces of constructible sheaves with microsupport in a Legendrian link.

We show that the "top open positroid variety" has mixed Hodge polynomial given by the q,t-rational Catalan numbers (up to a simple factor). The q,t-rational Catalan numbers satisfy remarkable symmetry and unimodality properties, and we show that these follow from the curious Lefschetz phenomenon for cluster varieties. The cohomologies of open positroid varieties are shown to be related to Khovanov–Rosanzky knot homology.

This talk is based on joint work with Pavel Galashin.

Newton–Okounkov bodies arising from cluster structures and mutations on polytopes

by Naoki Fujita (The University of Tokyo)

10:00–10:50 KST (UTC+9)

17:00–17:50 PDT (-1day) | 20:00–20:50 EDT (-1day) | 14:00–14:50 NZST

Abstract A toric degeneration is a flat degeneration from a projective variety to a toric variety, which can be used to apply the theory of toric varieties to other projective varieties. In this talk, we discuss relations among the following three constructions of toric degenerations: representation theory, Newton–Okounkov bodies, and cluster algebras. More precisely, we construct Newton–Okounkov bodies using cluster structures, and realize representation-theoretic and cluster-theoretic toric degenerations using this framework. We also discuss its relation with combinatorial mutations which was introduced in the context of mirror symmetry for Fano varieties. More precisely, we relate Newton–Okounkov bodies of flag varieties arising from cluster structures by combinatorial mutations. This talk is based on joint works with Hironori Oya and Akihiro Higashitani.

SL_3-laminations as bases for PGL_3 cluster varieties for surfaces

by Hyun Kyu Kim (Ewha Womans University)

11:00–11:50 KST (UTC+9)

18:00–18:50 PDT (-1day) | 21:00–21:50 EDT (-1day) | 15:00–15:50 NZST

Abstract I will recall Fock–Goncharov's duality conjecture for cluster $\mathcal{A}$- and $\mathcal{X}$-varieties, and Fock–Goncharov's solution for the case of certain enhanced moduli spaces of G-local systems on a punctured surface when G is SL_2 and PGL_2. Then I will explain how Kuperberg's web can be used to extend this result to the case when G is SL_3 and PGL_3.

January 14

Mirror symmetry through perverse schobers

by Benjamin Gammage (Havard University)

09:00–09:50 KST (UTC+9)

16:00–16:50 PDT (-1day) | 19:00–19:50 EDT (-1day) | 13:00–13:50 NZST

Abstract We explain how the language of perverse schobers gives a natural tool for describing a generalization of the Seidel–Sheridan strategy for computing Fukaya categories to the non-Lefschetz situation. We apply this technique to calculate the Fukaya category of the Milnor fiber of a Berglund–Hübsch singularity, building on some earlier computations of David Nadler. This calculation proves a conjecture of Lekili–Ueda.

Tropical Lagrangian multi-sections and smoothing of locally free sheaves on log Calabi–Yau surfaces

by Yat-Hin Suen (IBS Center for Geometry and Physics)

10:00–10:50 KST (UTC+9)

17:00–17:50 PDT (-1day) | 20:00–20:50 EDT (-1day) | 14:00–14:50 NZST

Abstract Homological mirror symmetry suggests that Lagrangian multi-sections over an integral affine manifold with singularities B should mirror to holomorphic vector bundles. In this talk, I will introduce the tropical version of Lagrangian multi-sections, called tropical Lagrangian multi-sections. I will mainly focus on dimension 2. To certain tropical Lagrangian multi-sections over B, I will construct a locally free sheaf E_0 on the log Calabi–Yau surface $X_0(B)$ associated to B and study the smoothability of the pair $(X_0(B), E_0)$. This is a joint work with Kwokwai Chan and Ziming Ma.

Orbifold Jacobian algebras and generalized Kodaira–Spencer maps

by Sangwook Lee (Soongsil University)

11:00–11:50 KST (UTC+9)

18:00–18:50 PDT (-1day) | 21:00–21:50 EDT (-1day) | 15:00–15:50 NZST

Abstract Given an algebraic function, its Jacobian algebra encodes the information of the singularity. There is also a notion of orbifold Jacobian algebras for functions which admit finite (abelian) group actions. We give a construction of an orbifold Jacobian algebra as Floer cohomology of a Lagrangian submanifold which represents homological mirror functor. We also discuss generalized Kodaira–Spencer maps whose image is not necessarily an ordinary Jacobian algebra. This talk is based on the joint work with C.-H. Cho.

January 15

Cluster coordinates from sheaf quantization of spectral curve

by Tatsuki Kuwagaki (Osaka University)

09:00–09:50 KST (UTC+9)

16:00–16:50 PDT (-1day) | 19:00–19:50 EDT (-1day) | 13:00–13:50 NZST

Abstract A sheaf quantization is a sheaf associated to a Lagrangian brane. In this talk, I will explain my construction of sheaf quantization of the spectral curves of Schrodinger equations, which is a part of conjectural $\hbar$-Riemann–Hilbert correspondence. The construction is based on exact WKB analysis. I will also explain an application to cluster theory. Iwaki–Nakanishi have found cluster variables in exact WKB analysis. The construction of sheaf quantization gives a geometric explanation of Iwaki–Nakanishi's cluster variables and their variants. A part of this talk is based on my joint work in progress with T. Ishibashi.

Dimers and Mirror Moduli

by Eric Zaslow (Northwestern University)

10:00–10:50 KST (UTC+9)

17:00–17:50 PDT (-1day) | 20:00–20:50 EDT (-1day) | 14:00–14:50 NZST

Abstract I will try to describe a counting problem that arises from considering mirror approaches to dimer integrable systems. Some of this talk is based on joint work with David Treumann and Harold Williams, and some is an ongoing project with Helge Ruddatand others.

Timetable: Conference

	08:50–09:00	09:00–09:50	10:00–10:50	11:00–11:50
Jan 11 (Mon)	Greetings	L. Ng	R. Casals	H. Gao
Jan 12 (Tue)		B. An	L. Shen	D. Weng
Jan 13 (Wed)		T. Lam	N. Fujita	H. Kim
Jan 14 (Thu)		B. Gammage	Y. Suen	S. Lee
Jan 15 (Fri)		T. Kuwagaki	E. Zaslow	

Local time

KST (UTC+9)	08:50	09:00–09:50	10:00–10:50	11:00–11:50
PDT (UTC-8)	15:50 (-1day)	16:00–16:50 (-1day)	17:00–17:50 (-1day)	18:00–18:50 (-1day)
EDT (UTC-5)	18:50 (-1day)	19:00–19:50 (-1day)	20:00–20:50 (-1day)	21:00–21:50 (-1day)
NZST (UTC+13)	12:50	13:00–13:50	14:00–14:50	15:00–15:50

Contact

`http://cgp.ibs.re.kr/conferences/LCM2021`

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